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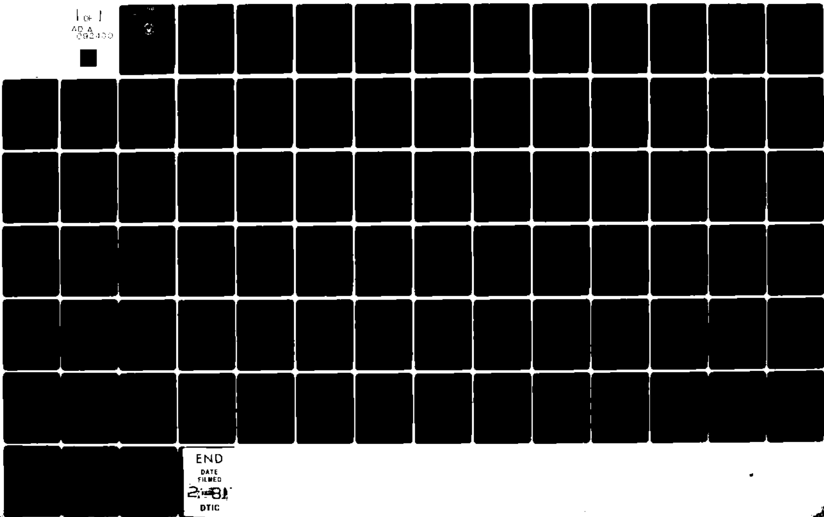

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APPLICATION OF NUMERICAL OPTIMIZATION
TO DRIVE SHAFT DESIGN

by

VIRGILIO S. MERCED

June 1980

Thesis Advisor: G. N. Vanderplaats

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Application of Numerical Optimization to Drive Shaft Design

by

Virgilio S. Merced
Lieutenant, United States Coast Guard

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

Naval Postgraduate School
June 1980

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ABSTRACT

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I. INTRODUCTION

Optimization and composite materials are two fields that have seen considerable developments in recent years. Advancements in these technologies have provided the design engineer with new dimensions, the depths of which are yet to be explored.

The major material characteristics that are advantageously cultivated in composites are the high strength and stiffness to weight ratios. The weight reduction is so significant that the aircraft and aerospace industries have concentrated a major portion of their reserach efforts in this field. Investigations in improving the resistance of composites against environmental factors such as temperatures, corrosion, and wear are continuing and findings affirm the contention that the future of this field is bright. However, other industries are slow in utilizing these materials in basic machinery elements, mainly due to a major drawback--the high cost of manufacturing and processing. It is in this vein that the importance of an optimized design becomes apparent. Advancements in fabrication and processing of composite materials coupled with the capability to optimize designs may well lead composites into a more competitive market.

With the ultimate goal of optimizing the design of a drive shaft made of composite materials, a research program was initiated. This report is the first in this investigative effort.

Before the intricacies introduced by the directional material properties of composites are included in the analysis, it was deemed necessary to probe the characteristics of the design of an isotropic material. This will enable one to foresee the trends and the adaptive measures necessary when the composite material properties are considered. The application of numerical optimization techniques to the design of an isotropic drive shaft is demonstrated here. The effects of a small mass imbalance on the bending moment and deflection of a shaft with synchronous whirl are investigated. Mainly, however, the objective is to develop an analysis procedure that may be adapted to the peculiarities of composites and ultimately be used in optimizing a composite drive shaft design.

Section 2 defines the scope and limitations of the analysis and optimization.

Section 3 discusses the analysis itself, the equations and the modes of failure that were considered. Examples of the analysis are shown.

Section 4 briefly describes the optimization program [1,2] and presents the results of optimizing a number of

shaft designs. Two variable function space diagrams are provided to illustrate the design space for different load conditions.

Section 5 contains recommendations for future investigations.

Appendix A contains a description of the FORTRAN program used in the analysis and the program text. Appendix B shows the derivation of equations for deflection, moment and shear force.

II. SCOPE AND LIMITATIONS

The analysis applies to shafts made of isotropic materials with simply supported or fixed-fixed end conditions. Basic shaft design formulas are used except that the deflection and moment equations are derived to include the effects of a small mass imbalance. The shaft is assumed to rotate with synchronous whirl.

To insure a continuous design field, the optimization is constrained to subcritical speeds only. Supercritical speeds introduce feasible design regions that are disjoint from the primary design space. In this investigation, these secondary areas are considered infeasible.

Since the optimization tends to a large radius-small thickness design, radial stresses are neglected. However, buckling of thin cylinders due to torsion and compression is considered. The equations used for these failure criteria do not include dynamic effects. A constant torque and a steady, uniform axial force are the loads considered in the formulas [3].

The analysis incorporates the capability to design a shaft that may be used in two or more loading conditions. For example, a shaft may be designed to transmit 50 HP at 300 RPM with an axial load of 2000 lbs. as well as to transmit 150 HP at 3000 RPM with a 1000 lb. axial load.

III. ANALYSIS

The problem is a shaft required to transmit a specified horsepower at a given speed. There may be an axial load, F or an internal presence, P. (See Figure 1.) Basic equations are used throughout the analysis and fundamentals such as formulas for area, volume, moment of inertia, polar moment of inertia and radius of gyration are not repeated here. However, certain stress calculations will be emphasized to illustrate the flow of analysis.

Appendix B shows the derivations of the equations for bending moment, deflection and shear for pinned-pinned and clamped-clamped end conditions. These derivations have taken into consideration the effects of a small mass imbalance in conjunction with the rotation of a shaft with synchronous whirl.

The inch-pound-second system of units (IPS) is used in the computer program analysis. However, by changing the equation for torque (line 50 in the FORTRAN text, Appendix A) to reflect the International System of Units, the program may be used effectively with consistent SI units.

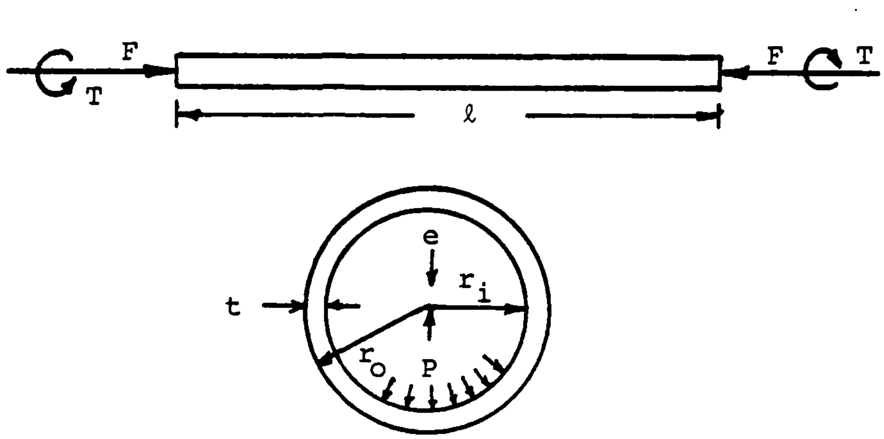
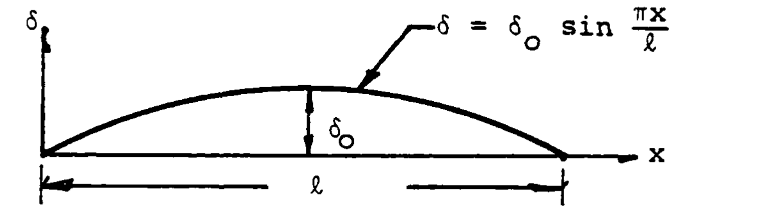
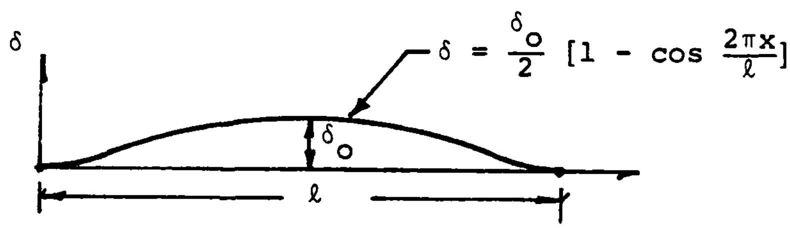


Figure 1. Applied Loads: Axial load (F), Internal pressure (P), Torque (T), Mass imbalance (e).



a. Pinned-pinned end condition



b. Clamped-clamped end condition

Figure 2. Assumed deflected shape of the shaft.

A. SIMPLY SUPPORTED BEAM

The deflected shape of the beam is assumed as:

$\delta = \delta_o \sin \frac{\pi x}{\ell}$ (see Fig. 2). The solution of the governing equations equations is given in Appendix B. The deflection is maximum at $x = \ell/2$

$$\delta_{max} = \frac{5\ell^4}{384} \left[\frac{K_1 e + K_2}{EI - K_1 (\frac{\ell}{\pi})^4 - F(\frac{\ell}{\pi})^2} \right] \qquad (3.1a)$$

where: $K_1 = \rho A \omega^2$

ρ = mass per unit volume

A = cross sectional area

ω = shaft speed is radians per second

K = ρAg = weight per unit length

e = eccentricity of mass with respect to the axis of rotation

ℓ = shaft length

E = Young's Modulus

I = Moment of Inertia

F = Axial Load

Certain characteristics of this equation are of particular interest. If there is no rotation nor axial load, the formula reduces to the classical equation for maximum beam deflection.

$$\delta_{max} = \frac{5K_2 \ell^4}{384EI} \qquad (3.1b)$$

If there is no axial load, the equation becomes unstable (deflection goes to infinity) when $EI = \rho A\omega^2 (\frac{\ell}{\pi})^4$ or, solving for the critical speed,

$$\omega_c = \pi^2 \sqrt{\frac{EI}{\rho A \ell^4}} \tag{3.1c}$$

which defines the first fundamental frequency of a simply supported shaft.

If there is no rotation, instability arises when $EI = F(\frac{\ell}{\pi})^2$ or

$$F_c = \frac{\pi^2 EI}{\ell^2} \tag{3.1d}$$

recognizable as Euler's column buckling criteria.

With both rotation and axial load, the critical speed becomes

$$\omega_c = \pi^2 \sqrt{\frac{EI - F(\frac{\ell}{\pi})^2}{\rho A \ell^4}} \tag{3.1e}$$

To avoid the instability regions, the denominator of equation (3.1a) must be given a lower bound greater than zero (point A in Fig. 3). Additionally, a maximum deflection has to be imposed to prevent computer overflow (point B in Fig. 3). These limits have no relevance with the deflection constraint used in the design optimization process (point C in Fig. 3), but are used only to prevent numerical ill-conditioning on the computer.

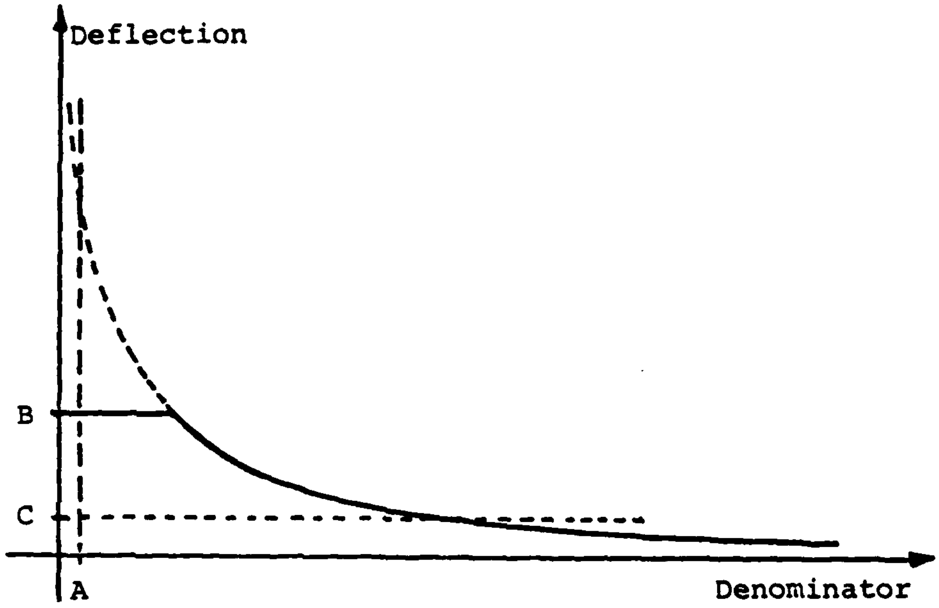


Figure 3. Limits on the deflection. The solid curve represents the deflection values used in the analysis.

The bending moment corresponding to the deflection calculated by equation (3.1a) at $x = \ell/_2$ is

$$M_{max} = K_1 \left[\frac{e}{8} + \delta \left(\frac{\ell}{\pi}\right)^2 \right] + F\delta + \frac{K_2 \ell^2}{8} \tag{3.1f}$$

When the weight tends to cancel the deflection, the corresponding moment at the same cross section becomes

$$M_{min} = M_{max} - \frac{K_2 \ell^2}{4} \tag{3.1g}$$

The shear force is maximum at x = 0.

$$V_{max} = \frac{K_1 \delta \ell}{\pi} + \frac{K_1 e \ell}{2} + \frac{K_2 \ell}{2} \tag{3.1h}$$

B. CLAMPED-CLAMPED BEAM

The deflected shape of the beam is assumed as:

$\delta = \frac{\delta_o}{2} \left[1 - \cos \frac{2\pi x}{\ell} \right]$. (See Fig. 2b.) The deflection is maximum at $x = \ell/_2$.

$$\delta = \ell^4 \left[\frac{K_1 e + K_2}{384\ EI - 0.746\ K_1 \ell^4 - 96 \left(\frac{\ell}{\pi}\right)^2 F} \right] \tag{3.2a}$$

As in the simply supported beam, similar observations regarding special loading conditions are apparent. If there is neither rotation nor axial load, the formula for maximum beam deflection results

$$\delta_{max} = \frac{K_2 \ell^4}{384\ EI} \tag{3.2b}$$

With no axial load, instability occurs when

$384\ EI = 0.746\ \rho A \omega^2$ or

$$\omega_c = 22.68 \sqrt{\frac{EI}{\rho A \ell^4}} \qquad (3.2c)$$

which is the first fundamental frequency for a fixed-fixed beam based on the assumed deflection used here.

When there is no rotation, instability arises when

$384\ EI = 96\ \frac{F\ell^2}{\pi^2}$ or

$$F_c = \frac{4\pi^2 EI}{\ell^2}, \qquad (3.2d)$$

Euler's column buckling criteria.

With rotation and axial load, the instability occurs at a smaller speed than that given by equation (3.2c),

$$\omega_c = 22.68 \sqrt{\frac{EI - 0.02533\ F\ell^4}{\rho A \ell^4}} \qquad (3.2e)$$

The bending moment at $x = \ell/_2$ corresponding to the deflection given by equation (3.2a) is

$$M_{max} = \frac{K_1 \ell}{24} \left[\frac{\delta \ell}{2} + \frac{3\delta}{\pi} + e\ell \right] + F\delta + \frac{K_2 \ell^2}{24} \qquad (3.2f)$$

A lesser bending moment occurs at the same cross section when the weight tends to cancel the deflection.

$$M_{min} = M_{max} - \frac{K_2 \ell^2}{12} \tag{3.2g}$$

The maximum shear occurs at x = 0.

$$V_{max} = \frac{K_1 \delta \ell}{4} + \frac{K_1 e \ell}{2} + \frac{K_2 \ell}{2} \tag{3.2h}$$

The preceding calculations are performed in Subroutine BEND, given in Appendix A.

C. STRESS ANALYSIS

With the deflection, shear and bending moments known at the critical cross sections, the analysis proceeds with the calculation of the stresses at the critical stress elements. At midspan, the critical stress elements are on the outer surface of the shaft where the axial stress (tensile or compressive) is maximum (Points A and B, Fig. 4). Since the shaft is in synchronous whirl, the same stress element, A, will generally tend to be in tension throughout the rotation, with the magnitude of the axial stress varying as the weight effect becomes alternatingly additive and subtractive. Element A is in the direction of the mass eccentricity from the center of the shaft while element B is on the outer surface in the opposite direction.

Most of the following equations are found in basic design and applied mechanics books. References will be noted when necessary.

Axial stress due to bending moment (tension/compression)

$$\sigma_x \text{ (max)} = \frac{M_{max} r_o}{I} \tag{3.3a}$$

$$\sigma_x \text{(min)} = \frac{M_{min} r_o}{I}$$

Axial stress due to axial load (compression)

$$\sigma_x = \frac{F}{A} \tag{3.3b}$$

The axial stress due to internal pressure for thin-wall cylinders is:

$$\sigma_x = \frac{Pr_i}{2t} \tag{3.3c}$$

Superposition with careful regard to sign conventions provides the maximum and minimum tensile and compressive axial stresses.

Hoop stress is primarily due to internal pressure and rotational inertia [4]

$$\sigma_H = \frac{Pr_o}{t} \tag{3.3d}$$

$$\sigma_H = \frac{\gamma w^2 r_o^2}{g} \cdot \frac{3+\nu}{4} \left[1 + \left(\frac{1-\nu}{3+\nu}\right)\left(\frac{r_i}{r_o}\right)^2\right] \tag{3.3e}$$

where γ = weight per unit volume

ω = rotation in radians per second

ν = Poisson's ratio

r_i = inside radius

r_o = outside radius

g = acceleration due to gravity

The shear stress at x - $\ell/_2$ is caused only by torsion (shear force is zero).

$$\tau_T = \frac{Tr_o}{J} \tag{3.3f}$$

where τ_T = shear stress due to torsion

T = torque

J = polar moment of inertia

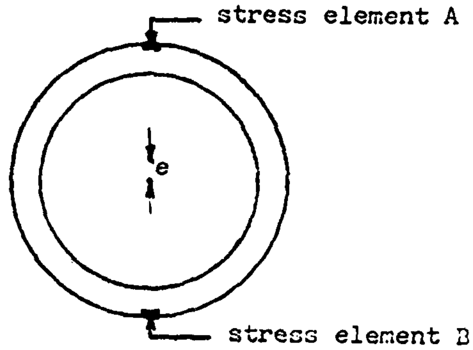
At x = 0, the critical element is where the shear stress due to the shear force along the cross section is additive with respect to the shear stress due to torsion (element C, Fig. 3). The shear stress due to V_{max} for thin wall cylinders [5] is

$$\tau_V = \frac{2\ V_{max}}{A} \tag{3.3g}$$

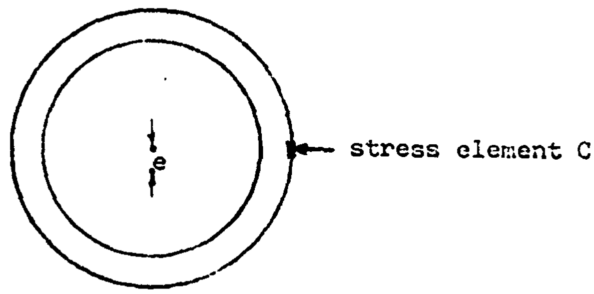
Superposition of equations (3.3f) and (3.3g) gives the total shear stress at the critical stress element.

The axial stress at element C due to bending moment is zero. The axial stress at this point is, therefore, caused only by the axial load and the internal pressure, equations (3.3b) and (3.3c).

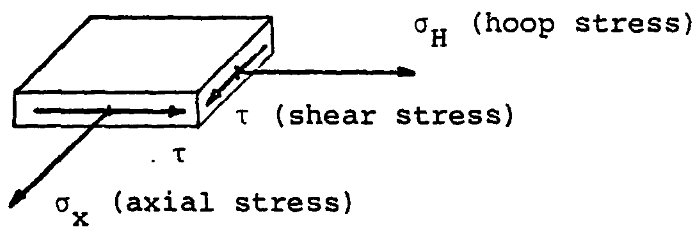
The hoop stress is calculated by equations (3.3d) and (3.3e).



a) At midspan



b) At $x = 0$



c) Typical stress element. Radial stresses are neglected.

Figure 4. Critical Stress Elements

D. MODES OF FAILURE

Eight modes of failure are considered in this investigation. The first two evaluate the static strengths at the critical stress elements using Von Mises criteria [5]. At midspan,

$$\sigma'_{mid} = \sqrt{\sigma_H^2 - \sigma_H \sigma_x + \sigma_x^2 + 3\tau_T^2} \tag{3.4a}$$

At x = 0

$$\sigma'_o = \sqrt{\sigma_H^2 - \sigma_H \sigma_x + \sigma_x^2 + 3\tau_{T+V}^2} \tag{3.4b}$$

Failure is assumed if the yield strength of the material is equal to or less than these Von Mises stresses multiplied by a safety factor.

A limit on the deflection is imposed. A common practice is to specify a maximum deflection in inch per foot of shaft length. For example, as used in this report,

$$\delta_{max} = 0.005 \text{ inch/foot length}$$

Since the optimization may lead to a thin cylinder design, buckling due to torsion and compression are considered. The critical torque and critical compressive stress for buckling of thin cylinders [3] are;

$$T_c = \frac{\pi\sqrt{2}\,E}{3(1-\nu^2)^{0.75}} \cdot \sqrt{rt^5} \tag{3.4c}$$

$$\sigma_c = \frac{Et}{r\sqrt{3(1-\nu^2)}} \qquad (3.4d)$$

Experimental data indicate that actual failure of the cylinders usually occurs below fifty percent of the values calculated from these equations. To accommodate this discrepancy and the fact that the dynamic effects are not considered in the derivation of these equations, comparatively higher safety factors must be used for these failure modes.

The shaft is also checked for column buckling using Johnson's or Euler's equations [5] as applicable. The actual slenderness ratio is compared with the slenderness ratio where transition occurs to determine which formulas apply. The calculated critical load, using the appropriate equations for the end condition, is compared to the actual load, again incorporating a safety factor. Equations (3.1d) or (3.2d) are used if the slenderness ratio falls in the Euler range. In the Johnson region, the following equations [5] apply.

$$\frac{F_c}{A} = S_c - b\left(\frac{\ell}{K}\right)^2 \qquad (3.4e)$$

where S_c = yield strength in compression

$(\frac{\ell}{K})$ = slenderness ratio

$b = \left[\frac{S_c}{2\pi}\right]^2 \frac{1}{nE}$

E = Young's Modulus

n = 1 for simple supports

4 for fixed-fixed ends

The effects of the fluctuating axial stresses are evaluated for fatigue failure using the Goodman diagram [5]. Von Mises stresses for the mean and alternating stresses are calculated as follows:

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \tag{3.4f}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \tag{3.4g}$$

$$\sigma_m' = \sqrt{\sigma_H^2 - \sigma_H \sigma_m + \sigma_m^2 + 3 \tau_T^2} \tag{3.4h}$$

$$\sigma_a' = \sigma_a \tag{3.4i}$$

The Goodman diagram is drawn (Fig. 5) and the safety factor is determined analytically.

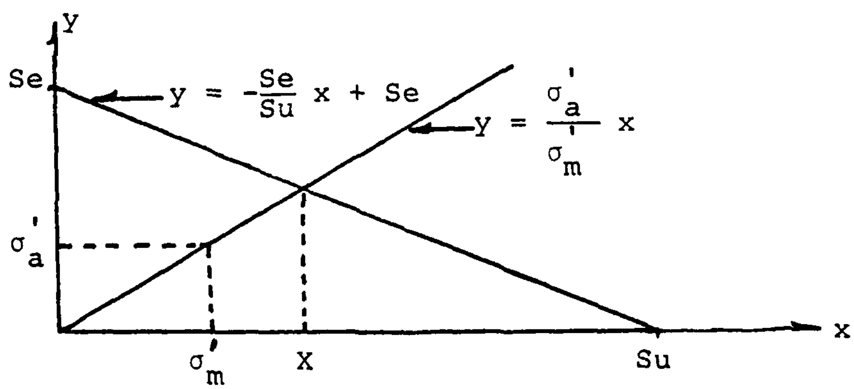


Figure 5. Goodman Diagram

$$SF = \frac{X}{\sigma_m'} = \frac{Se}{\sigma_m' \left[\frac{\sigma_a'}{\sigma_m'} + \frac{Se}{Su}\right]} \tag{3.4j}$$

where Se = fully corrected endurance limit

Su = ultimate strength

This safety factor is required to be greater than a specified value.

Lastly, failure is assumed if a specified percentage of the critical speed as calculated by equations (3.1e) or (3.2e) is less than the shaft speed.

E. ANALYSIS EXAMPLES

The examples are chosen to illustrate several aspects of the shaft design problem. The first is a design which violates almost all of the constraints while the second example is one within the feasible region of the design field.

The material used in all the examples in this report is UN G10150 HR steel, the properties of which are listed in Table I [5].

TABLE I

MATERIAL PROPERTIES

UNS G10150 HR STEEL

Young's Modulus	30.0E 06 psi
Shear Modulus	11.5E 06 psi
Poisson's Ratio	0.292
Weight Per Unit Volume	0.282 lbs/in^3
Yield Strength	27.0E 03 psi
Ultimate Strength	50.0E 03 psi

1. <u>Analysis Example 1</u>

A simply supported shaft with 0.1 inch eccentricity is to transmit 150 HP at 3000 RPM (3150 in-lb torque), with an axial load of 2000 lbs. For a shaft length of 120 inches, inside radius of 1.0 inch and thickness of 1.0 inch, the design violates the static and dynamic strength, deflection, buckling due to compression, and the speed constraints. The analysis of this design is given below where an asterisk (*) denotes violated constraints on the design.

Volume	0.11310E 04 in^3
Weight	0.31893E 03 lbs
Moment of Inertia	0.11781E 02 in^4
Polar Moment of Inertia	0.23562E 02 in^4
*Critical Speed	0.14767E 04 RPM
Critical Axial Load	0.18764E 06 lbs
Critical Buckling Torque	0.47469E 08 in lb
*Critical Buckling Stress	0.905493 07 psi
*Deflection	0.24000E 02 in
Bending Moment	0.36420E 08 in lb
Shear Force	0.62722E 06 lb
Axial Stress at A	0.95779E 07 psi
Axial Stress at B	-0.95783E 07 psi
Hoop Stress	0.25013E 03 psi
Torsional Shear Stress	0.26738E 03 psi
Total Shear Stress	0.13337E 06 psi
Mean Axial Stress	0.95771E 07 psi
Alternating Axial Stress	0.81250E 03 psi
*Von Mises at Midspan	0.95785E 07 psi
*Von Mises at x = 0	0.23100E 06 psi

The calculated deflection is the maximum value imposed to avoid numerical ill-conditioning (point B, Fig. 3). Since the deflection affects most of the relevant parameters in the analysis, the calculated stresses are unreliable and not valid. These values, however, tend to direct the design towards the feasible region.

2. Analysis Example 2

This shaft is similar to the first example in design conditions and loading. The only difference is that the dimensions are such that no constraints are violated, but two constraints are active or critical. The shaft is 120 inches long with an inside radius of 6.39 inches and thickness of 0.0369 inch. This, in fact, is the optimized design for the given loading as calculated by COPES-CONMIN [1,2]. The results of the analysis follows, where a double asterisk (**) denotes values at their design limit.

Volume	0.17866E 03 in^3
Weight	0.50382E 02 lbs
Moment of Inertia	0.30629E 02 in^4
Polar Moment of Inertia	0.61258E 02 in^4
**Critical Speed	0.60061E 04 RPM
Critical Axial Load	0.39557E 05 lbs
**Critical Buckling Torque	0.31490E 05 in lb
Critical Buckling Stress	0.10400E 06 psi
Deflection	0.43813E-01 in
Bending Moment	0.17026E 05 in lb
Shear Force	0.84895E 03 lbs
Axial Stress at A	0.22326E 04 psi
Axial Stress at B	-0.49193E 04 psi
Hoop Stress	0.29778E 04 psi
Torsional Shear Stress	0.33079E 03 psi
Total Shear Stress	0.14712E 04 psi
Mean Axial Stress	0.20739E 04 psi
Alternating Axial Stress	0.15872E 03 psi
Von Mises at Midspan	0.69313E 04 psi
Von Mises at x = 0	0.39193E 04 psi

In each of the examples, the analysis proceeds as follows. All shaft characteristics like cross sectional area, volume, weight, moment of inertia and polar moment of inertia are first calculated. Parameters that depend only on geometry such as critical torque and compressive stress for buckling of thin cylinders, and critical speed are then determined before Subroutine BEND, where the deflection, bending moments and shear force at the critical cross sections are computed, is called.

At midspan, the absolute values of the axial stresses at elements A and B (see Fig. 4a) are compared to determine which element is critical. In both examples, element B is considered critical for strength evaluation. At $x = 0$, the critical element, C, is where the torsional shear stress and the shear stress due to the shear force along the cross section are additive. The Von Mises stresses for each of the critical elements are then calculated using equations (3.4a) and (3.4b). The rest of the analysis is a straight forward application of the equations for the critical parameters and the constraints.

IV. OPTIMIZATION, RESULTS AND CONCLUSIONS

A. OPTIMIZATION AND RESULTS

There are a number of optimization programs available, each one using different techniques in locating the desired optimum design. COPES-CONMIN [1,2] is a versatile program that may be used for sensitivity analysis and two variable function space study as well as an optimization tool. Provided with a user supplied analysis program (Subroutine ANALIZ) where the objective function, constraints and other relevant parameters are calculated, it determines a usable and feasible sector from which a search direction is chosen. This choice of search direction is made by using such information as the gradients of the objective function, gradients of active and violated constraints and the degree of push-off these constraints violations generate. This iterative process of minimizing/maximizing the objective function by changing the design variables is terminated when no further improvement can be made.

In this exercise, the objective function to be minimized is the volume, with the radius and thickness as design variables. Several design examples are shown to demonstrate the effects of the different parameters on the optimized design and on the design space itself. Seven examples are

presented corresponding to the loading conditions as listed in Table II. Two variable function space diagrams are generated to illustrate these effects. The numbered curves in the graphs represent the corresponding constraints shown in Table III. The shaded side of a curve represents the unfeasible area of the design space with respect to the corresponding constraint. The dashed curves represent contours of the objective function, the numbers indicating the volume in cubic inches.

TABLE II

OPTIMIZATION EXAMPLES

No.	Loads			
	Horsepower	RPM	Pressure (psi)	Axial Load (lbs)
1	150	300	0	0
2	150	300	1000	0
3	150	300	0	2000
4	150	3000	0	0
5	150	3000	1000	0
6	150	3000	0	2000
7*	150	300	0	0
	150	3000	0	2000

*Example No. 7 is a design for two load conditions.

TABLE III

DESIGN CONSTRAINTS

Number	Constraint
1	Static Strength at Midspan
2	Static Strength at x = 0
3	Deflection
4	Buckling due to Torque
5	Buckling due to Compression
6	Column Buckling
7	Dynamic Strength (Fatigue)
8	Speed

EXAMPLE 1. : INITIAL DESIGN

SHAFT DIMENSIONS :

RI = 0.10000E 01 TH = 0.10000E 01 SL = 0.12000E

DESIGN CONDITIONS :

ECCENTRICITY = 0.10000E 00

END CONDITIONS : PINNED-PINNED

LOADS :

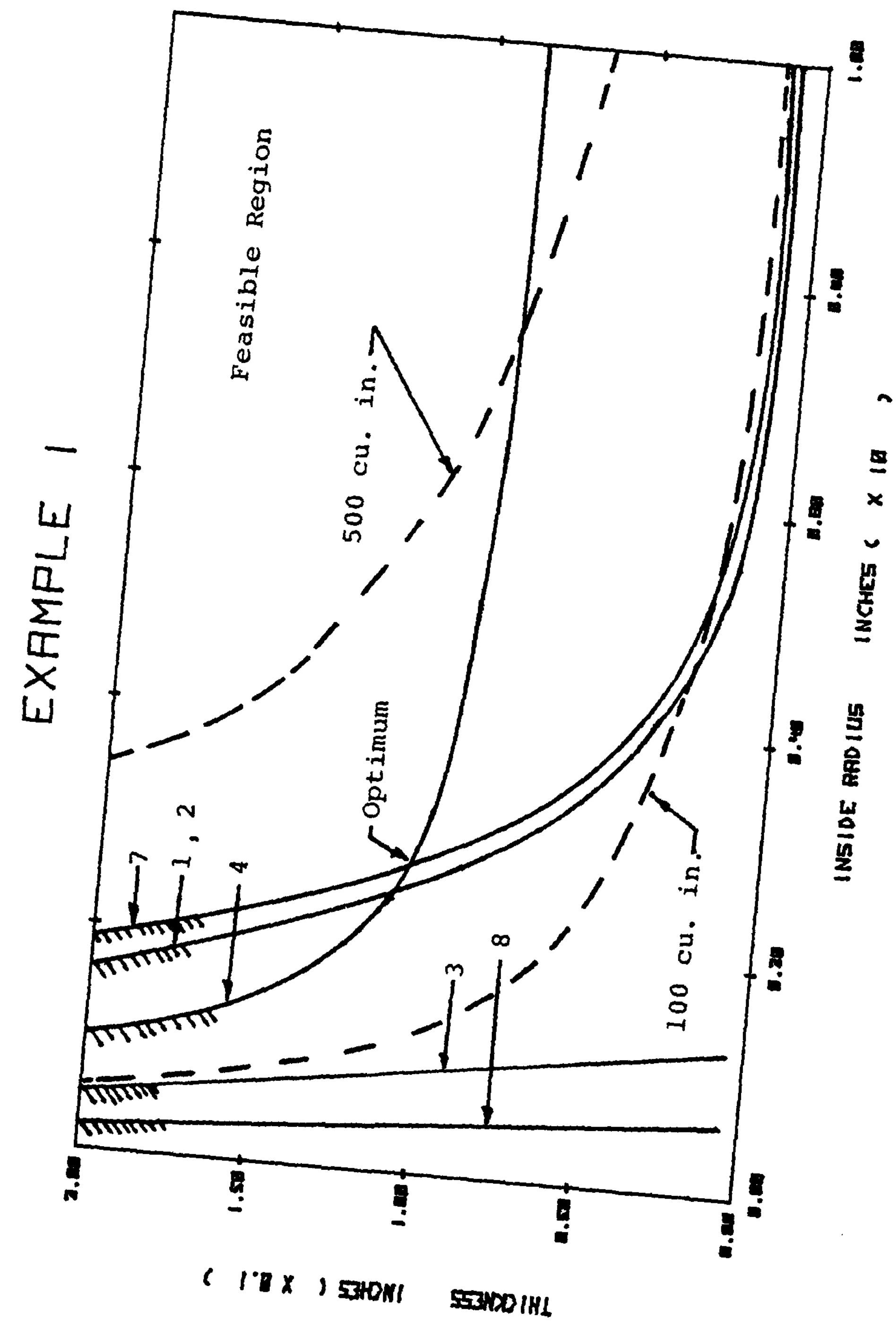
HORSEPOWER	RPM	PRESSURE	AXIAL LOAD
0.15000E 03	0.30000E 03	0.0	0.0

ANALYSIS RESULTS

TORQUE(1)	0.31500E 05
VOLUME	0.11310E 04
WEIGHT	0.31893E 03
MOMENT OF INERTIA	0.11781E 02
POLAR MOMENT OF INERTIA	0.23562E 02
CRITICAL SPEED	0.14828E 04
CRITICAL AXIAL LOAD	0.18754E 06
CRITICAL BUCKLING TORQUE	0.47469E 08
CRITICAL BUCKLING STRESS	0.90549E 07
DEFLECTION	0.26584E-01
BENDING MOMENT	0.50477E 04
SHEAR FORCE	0.20714E 03
AXIAL STRESS AT A	0.85692E 03
AXIAL STRESS AT B	-0.85692E 03
HOOP STRESS	0.25013E 01
TORSIONAL SHEAR STRESS	0.26738E 04
TOTAL SHEAR STRESS	0.27178E 04
MEAN AXIAL STRESS	0.44760E 02
ALTERNATING AXIAL STRESS	0.81216E 03
VON MISES AT MIDSPAN	0.47095E 04
VON MISES AT X=0.	0.47073E 04

```
            EXAMPLE 1. :  OPTIMUM DESIGN
SHAFT DIMENSIONS :
 RI = 0.25847E 01   TH = 0.11124E 00   SL = 0.12000E
DESIGN CONDITIONS :
 ECCENTRICITY = 0.10000E 00
END CONDITIONS : PINNED-PINNED
LOADS :
HORSEPOWER       RPM          PRESSURE       AXIAL LOAD
 0.15000E 03  0.30000E 03  0.0            0.0

              ANALYSIS RESULTS
TORQUE(1)                             0.31500E 05
VOLUME                                0.22145E 03
WEIGHT                                0.62451E 02
MOMENT OF INERTIA                     0.64353E 01
POLAR MOMENT OF INERTIA               0.12871E 02
CRITICAL SPEED                        0.24757E 04
CRITICAL AXIAL LOAD                   0.45137E 05
CRITICAL BUCKLING TORQUE              0.31500E 06
CRITICAL BUCKLING STRESS              0.74729E 06
DEFLECTION                            0.92753E-02
BENDING MOMENT                        0.95479E 03
SHEAR FORCE                           0.39681E 02
AXIAL STRESS AT A                     0.39998E 03
AXIAL STRESS AT B                    -0.39998E 03
HOOP STRESS                           0.51655E 01
TORSIONAL SHEAR STRESS                0.65980E 04
TOTAL SHEAR STRESS                    0.66410E 04
MEAN AXIAL STRESS                     0.75507E 01
ALTERNATING AXIAL STRESS              0.39243E 03
VON MISES AT MIDSPAN                  0.11435E 05
VON MISES AT X=0.                     0.11503E 05
```


EXAMPLE I
Feasible Region
500 cu. in.
Optimum
100 cu. in.
7
1, 2
4
3
8
INSIDE RADIUS INCHES (X 10)
THICKNESS INCHES (X 0.1)

EXAMPLE 2. : INITIAL DESIGN

SHAFT DIMENSIONS :

RI = 0.10000E 01 TH = 0.10000E 01 SL = 0.12000E

DESIGN CONDITIONS :

ECCENTRICITY = 0.10000E 00

END CONDITIONS : PINNED-PINNED

LOADS :

HORSEPOWER	RPM	PRESSURE	AXIAL LOAD
0.15000E 03	0.30000E 03	0.10000E 04	0.0

ANALYSIS RESULTS

TORQUE(1)	0.31500E 05
VOLUME	0.11310E 04
WEIGHT	0.31893E 03
MOMENT OF INERTIA	0.11781E 02
POLAR MOMENT OF INERTIA	0.23562E 02
CRITICAL SPEED	0.14828E 04
CRITICAL AXIAL LOAD	0.18764E 06
CRITICAL BUCKLING TORQUE	0.47469E 08
CRITICAL BUCKLING STRESS	0.90549E 07
DEFLECTION	0.26584E-01
BENDING MOMENT	0.50477E 04
SHEAR FORCE	0.20714E 03
AXIAL STRESS AT A	0.18569E 04
AXIAL STRESS AT B	0.14308E 03
HOOP STRESS	0.66917E 03
TORSIONAL SHEAR STRESS	0.26738E 04
TOTAL SHEAR STRESS	0.27178E 04
MEAN AXIAL STRESS	0.10448E 04
ALTERNATING AXIAL STRESS	0.81216E 03
VON MISES AT MIDSPAN	0.49093E 04
VON MISES AT X=0.	0.47546E 04

EXAMPLE 2. : OPTIMUM DESIGN

SHAFT DIMENSIONS :

RI = 0.18625E 01 TH = 0.25560E 00 SL = 0.12000E

DESIGN CONDITIONS :

ECCENTRICITY = 0.10000E 00

END CONDITIONS : PINNED-PINNED

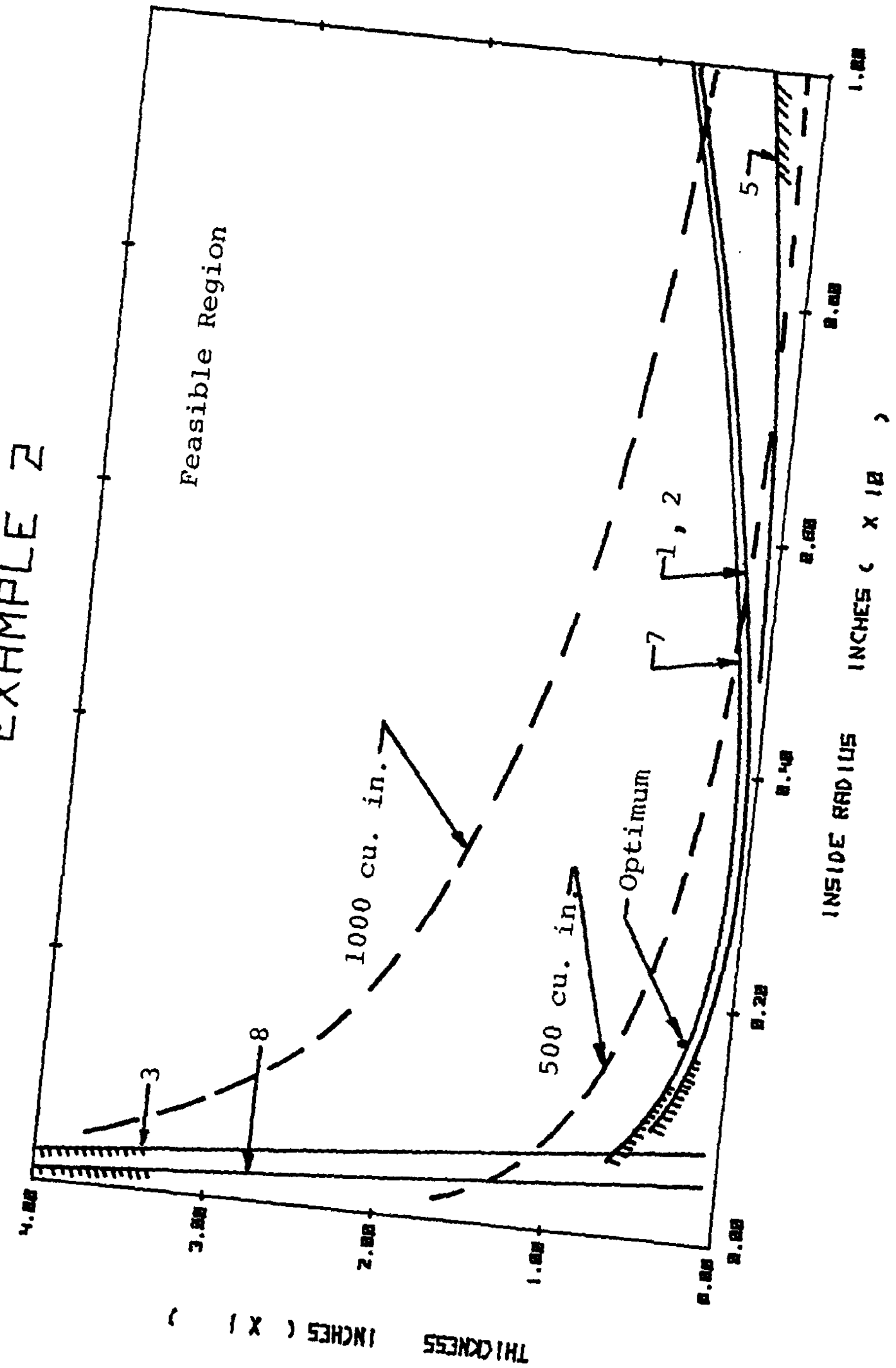
LOADS :

HORSEPOWER	RPM	PRESSURE	AXIAL LOAD
0.15000E 03	0.30000E 03	0.10000E 04	0.0

ANALYSIS RESULTS

TORQUE(1)	0.31500E 05
VOLUME	0.39958E 03
WEIGHT	0.11268E 03
MOMENT OF INERTIA	0.66578E 01
POLAR MOMENT OF INERTIA	0.13316E 02
CRITICAL SPEED	0.18754E 04
CRITICAL AXIAL LOAD	0.75144E 05
CRITICAL BUCKLING TORQUE	0.23553E 07
CRITICAL BUCKLING STRESS	0.22602E 07
DEFLECTION	0.16358E-01
BENDING MOMENT	0.17476E 04
SHEAR FORCE	0.72247E 02
AXIAL STRESS AT A	0.75709E 04
AXIAL STRESS AT B	0.64537E 04
HOOP STRESS	0.65487E 04
TORSIONAL SHEAR STRESS	0.50344E 04
TOTAL SHEAR STRESS	0.50779E 04
MEAN AXIAL STRESS	0.70306E 04
ALTERNATING AXIAL STRESS	0.54027E 03
VON MISES AT MIDSPAN	0.11254E 05
VON MISES AT X=0.	0.10965E 05

EXAMPLE 2



Feasible Region
1000 cu. in.
500 cu. in.
Optimum
1, 2
7
5
3
8
4.00
3.00
2.00
1.00
0.00
0.20
0.40
0.60
0.80
1.00
INSIDE RADIUS INCHES (X 10)
THICKNESS INCHES (X 1)

EXAMPLE 3. : INITIAL DESIGN

SHAFT DIMENSIONS :

RI = 0.10000E 01 TH = 0.10000E 01 SL = 0.12000E

DESIGN CONDITIONS :

ECCENTRICITY = 0.10000E 00

END CONDITIONS : PINNED-PINNED

LOADS :

HORSEPOWER	RPM	PRESSURE	AXIAL LOAD
0.15000E 03	0.30000E 03	0.0	0.20000E 04

ANALYSIS RESULTS

TORQUE(1)	0.31500E 05
VOLUME	0.11310E 04
WEIGHT	0.31893E 03
MOMENT OF INERTIA	0.11781E 02
POLAR MOMENT OF INERTIA	0.23562E 02
CRITICAL SPEED	0.14767E 04
CRITICAL AXIAL LOAD	0.18764E 06
CRITICAL BUCKLING TORQUE	0.47469E 08
CRITICAL BUCKLING STRESS	0.90549E 07
DEFLECTION	0.26814E-01
BENDING MOMENT	0.54144E 04
SHEAR FORCE	0.20720E 03
AXIAL STRESS AT A	0.70697E 03
AXIAL STRESS AT B	-0.11314E 04
HOOP STRESS	0.25013E 01
TORSIONAL SHEAR STRESS	0.26738E 04
TOTAL SHEAR STRESS	0.27178E 04
MEAN AXIAL STRESS	-0.10519E 03
ALTERNATING AXIAL STRESS	0.81216E 03
VON MISES AT MIDSPAN	0.47677E 04
VON MISES AT X=0.	0.47073E 04

EXAMPLE 3. : OPTIMUM DESIGN

SHAFT DIMENSIONS :

RI = 0.25907E 01 TH = 0.11119E 00 SL = 0.12000E

DESIGN CONDITIONS :

ECCENTRICITY = 0.10000E 00

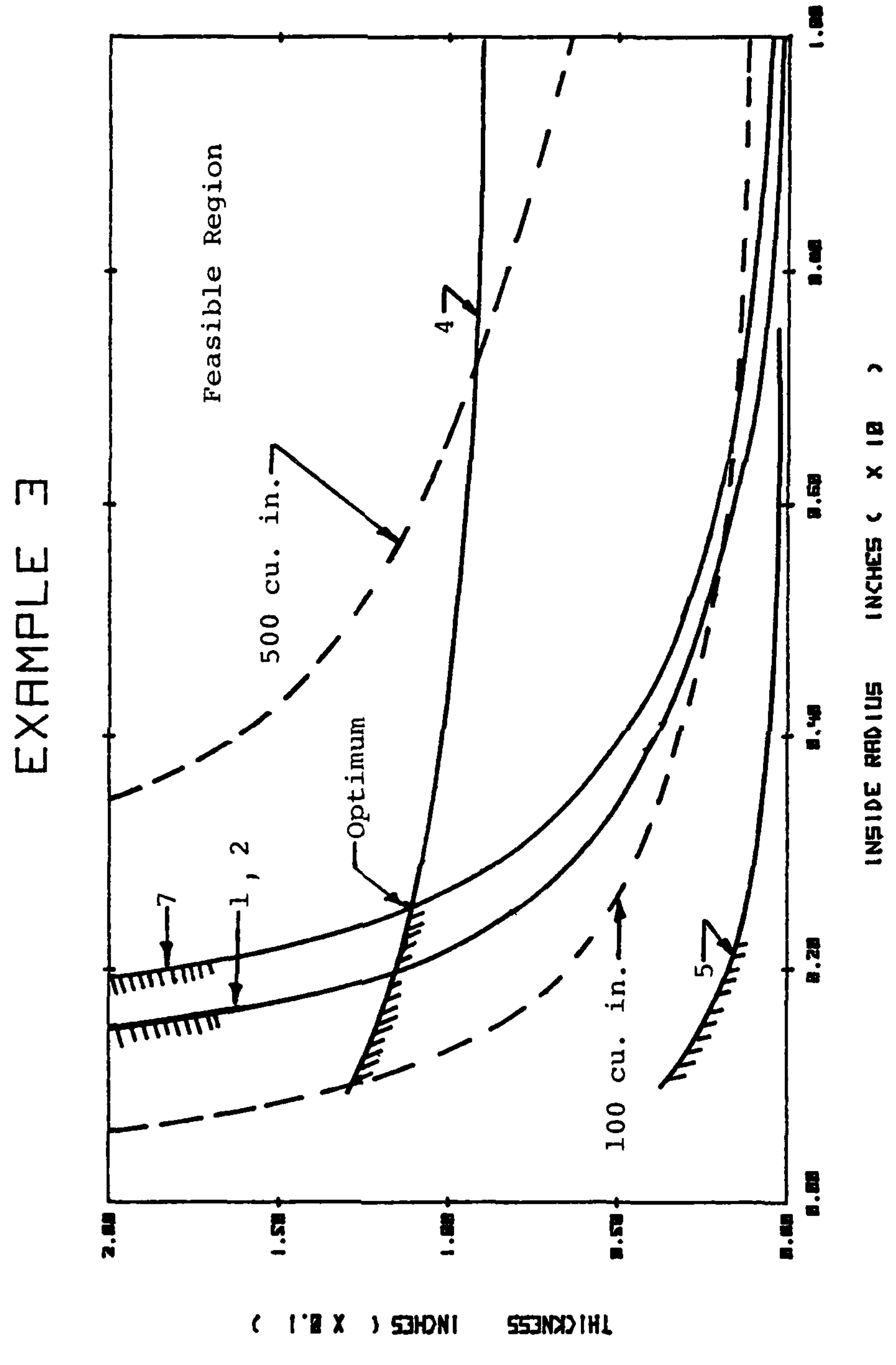
END CONDITIONS : PINNED-PINNED

LOADS :

HORSEPOWER	RPM	PRESSURE	AXIAL LOAD
0.15000E 03	0.30000E 03	0.0	0.20000E 04

ANALYSIS RESULTS

TORQUE(1)	0.31500E 05
VOLUME	0.22186E 03
WEIGHT	0.62564E 02
MOMENT OF INERTIA	0.64766E 01
POLAR MOMENT OF INERTIA	0.12953E 02
CRITICAL SPEED	0.24636E 04
CRITICAL AXIAL LOAD	0.45241E 05
CRITICAL BUCKLING TORQUE	0.31500E 06
CRITICAL BUCKLING STRESS	0.74527E 06
DEFLECTION	0.93752E-02
BENDING MOMENT	0.98171E 03
SHEAR FORCE	0.39758E 02
AXIAL STRESS AT A	-0.67221E 03
AXIAL STRESS AT B	-0.14913E 04
HOOP STRESS	0.51888E 01
TORSIONAL SHEAR STRESS	0.65706E 04
TOTAL SHEAR STRESS	0.66136E 04
MEAN AXIAL STRESS	-0.10637E 04
ALTERNATING AXIAL STRESS	0.39151E 03
VON MISES AT MIDSPAN	0.11478E 05
VON MISES AT X=0.	0.11455E 05



EXAMPLE 3
Feasible Region
500 cu. in.
4
Optimum
7
1 , 2
100 cu. in.
5
THICKNESS INCHES (X 0.1)
INSIDE RADIUS INCHES (X 10)

EXAMPLE 4. : INITIAL DESIGN

SHAFT DIMENSIONS :

RI = 0.10000E 01 TH = 0.10000E 01 SL = 0.12000E

DESIGN CONDITIONS :

ECCENTRICITY = 0.10000E 00

END CONDITIONS : PINNED-PINNED

LOADS :

HORSEPOWER	RPM	PRESSURE	AXIAL LOAD
0.15000E 03	0.30000E 04	0.0	0.0

ANALYSIS RESULTS

TORQUE(1)	0.31500E 04
VOLUME	0.11310E 04
WEIGHT	0.31893E 03
MOMENT OF INERTIA	0.11781E 02
POLAR MOMENT OF INERTIA	0.23562E 02
CRITICAL SPEED	0.14828E 04
CRITICAL AXIAL LOAD	0.18764E 06
CRITICAL BUCKLING TORQUE	0.47469E 08
CRITICAL BUCKLING STRESS	0.90549E 07
DEFLECTION	0.24000E 02
BENDING MOMENT	0.23801E 08
SHEAR FORCE	0.62722E 06
AXIAL STRESS AT A	0.40406E 07
AXIAL STRESS AT B	-0.40406E 07
HOOP STRESS	0.25013E 03
TORSIONAL SHEAR STRESS	0.26738E 03
TOTAL SHEAR STRESS	0.13337E 06
MEAN AXIAL STRESS	0.40397E 07
ALTERNATING AXIAL STRESS	0.81200E 03
VON MISES AT MIDSPAN	0.40404E 07
VON MISES AT X=0.	0.23100E 06

EXAMPLE 4. : OPTIMUM DESIGN

SHAFT DIMENSIONS :

RI = 0.63798E 01   TH = 0.36963E-01   SL = 0.12000E

DESIGN CONDITIONS :

ECCENTRICITY = 0.10000E 00

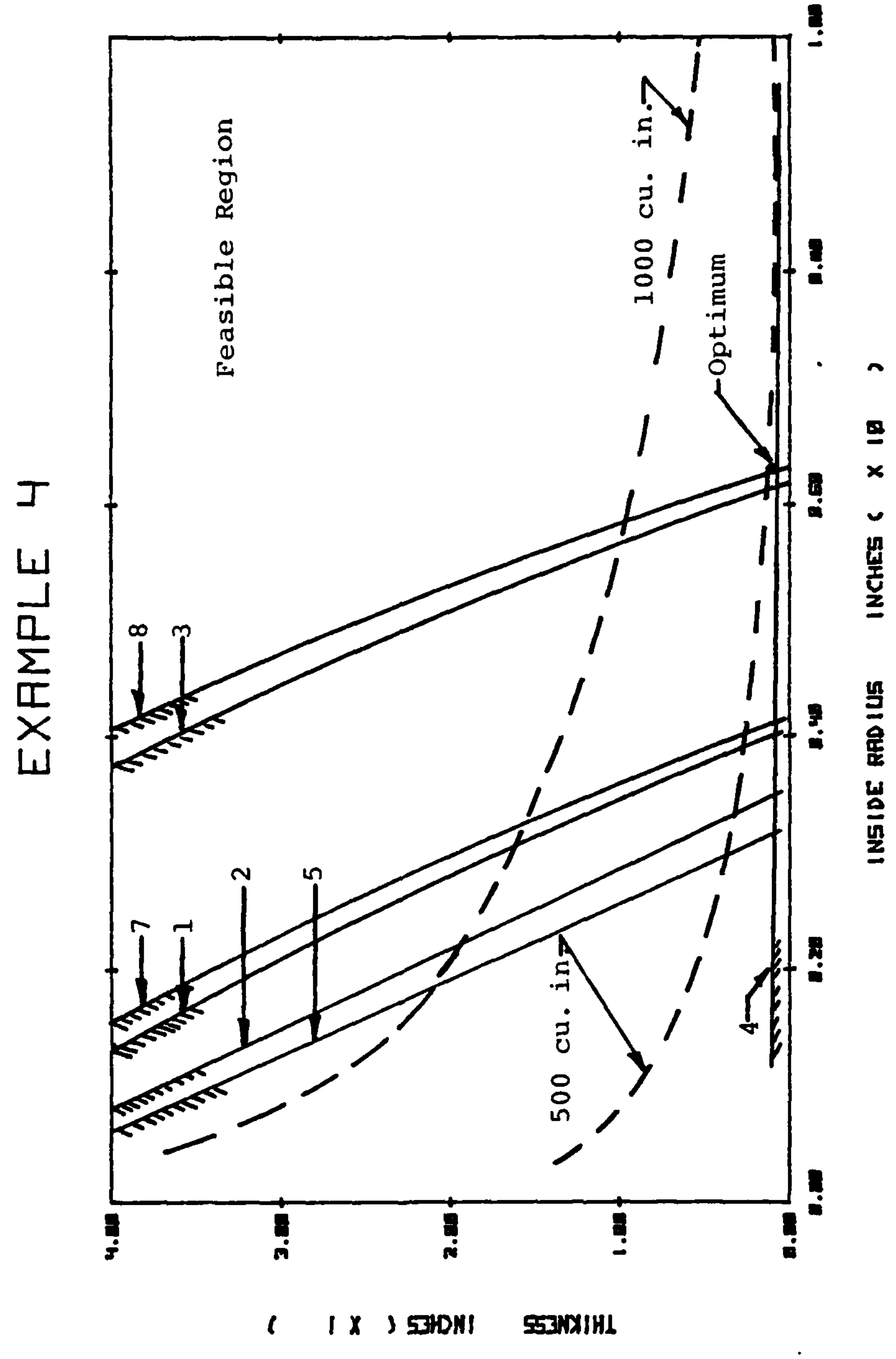
END CONDITIONS : PINNED-PINNED

LOADS :

HORSEPOWER	RPM	PRESSURE	AXIAL LOAD
0.15000E 03	0.30000E 04	0.0	0.0

ANALYSIS RESULTS

TORQUE(1)	0.31500E 04
VOLUME	0.17831E 03
WEIGHT	0.50283E 02
MOMENT OF INERTIA	0.30415E 02
POLAR MOMENT OF INERTIA	0.60830E 02
CRITICAL SPEED	0.60005E 04
CRITICAL AXIAL LOAD	0.39476E 05
CRITICAL BUCKLING TORQUE	0.31494E 05
CRITICAL BUCKLING STRESS	0.10432E 06
DEFLECTION	0.43922E-01
BENDING MOMENT	0.76214E 04
SHEAR FORCE	0.84773E 03
AXIAL STRESS AT A	0.16079E 04
AXIAL STRESS AT B	-0.16079E 04
HOOP STRESS	0.29628E 04
TORSIONAL SHEAR STRESS	0.33228E 03
TOTAL SHEAR STRESS	0.14733E 04
MEAN AXIAL STRESS	0.14488E 04
ALTERNATING AXIAL STRESS	0.15913E 03
VON MISES AT MIDSPAN	0.26327E 04
VON MISES AT X=0.	0.39103E 04



EXAMPLE 4
Feasible Region
1000 cu. in.
Optimum
500 cu. in.
8
3
7
1
2
5
4
INSIDE RADIUS INCHES (X 10)
THICKNESS INCHES (X 1)

EXAMPLE 5. : INITIAL DESIGN

SHAFT DIMENSIONS :

RI = 0.10000E 01 TH = 0.10000E 01 SL = 0.12000E

DESIGN CONDITIONS :

ECCENTRICITY = 0.10000E 00

END CONDITIONS : PINNED-PINNED

LOADS :

HORSEPOWER	RPM	PRESSURE	AXIAL LOAD
0.15000E 03	0.30000E 04	0.10000E 04	0.0

ANALYSIS RESULTS

TORQUE(1)	0.31500E 04
VOLUME	0.11310E 04
WEIGHT	0.31893E 03
MOMENT OF INERTIA	0.11781E 02
POLAR MOMENT OF INERTIA	0.23562E 02
CRITICAL SPEED	0.14828E 04
CRITICAL AXIAL LOAD	0.18764E 06
CRITICAL BUCKLING TORQUE	0.47469E 08
CRITICAL BUCKLING STRESS	0.90549E 07
DEFLECTION	0.24000E 02
BENDING MOMENT	0.23801E 08
SHEAR FORCE	0.62722E 06
AXIAL STRESS AT A	0.40416E 07
AXIAL STRESS AT B	-0.40396E 07
HOOP STRESS	0.91680E 03
TORSIONAL SHEAR STRESS	0.26738E 03
TOTAL SHEAR STRESS	0.13337E 06
MEAN AXIAL STRESS	0.40407E 07
ALTERNATING AXIAL STRESS	0.81200E 03
VON MISES AT MIDSPAN	0.40411E 07
VON MISES AT X=0.	0.23100E 06

EXAMPLE 5. : OPTIMUM DESIGN

SHAFT DIMENSIONS :

RI = 0.60844E 01 TH = 0.61266E 00 SL = 0.12000E

DESIGN CONDITIONS :

ECCENTRICITY = 0.10000E 00

END CONDITIONS : PINNED-PINNED

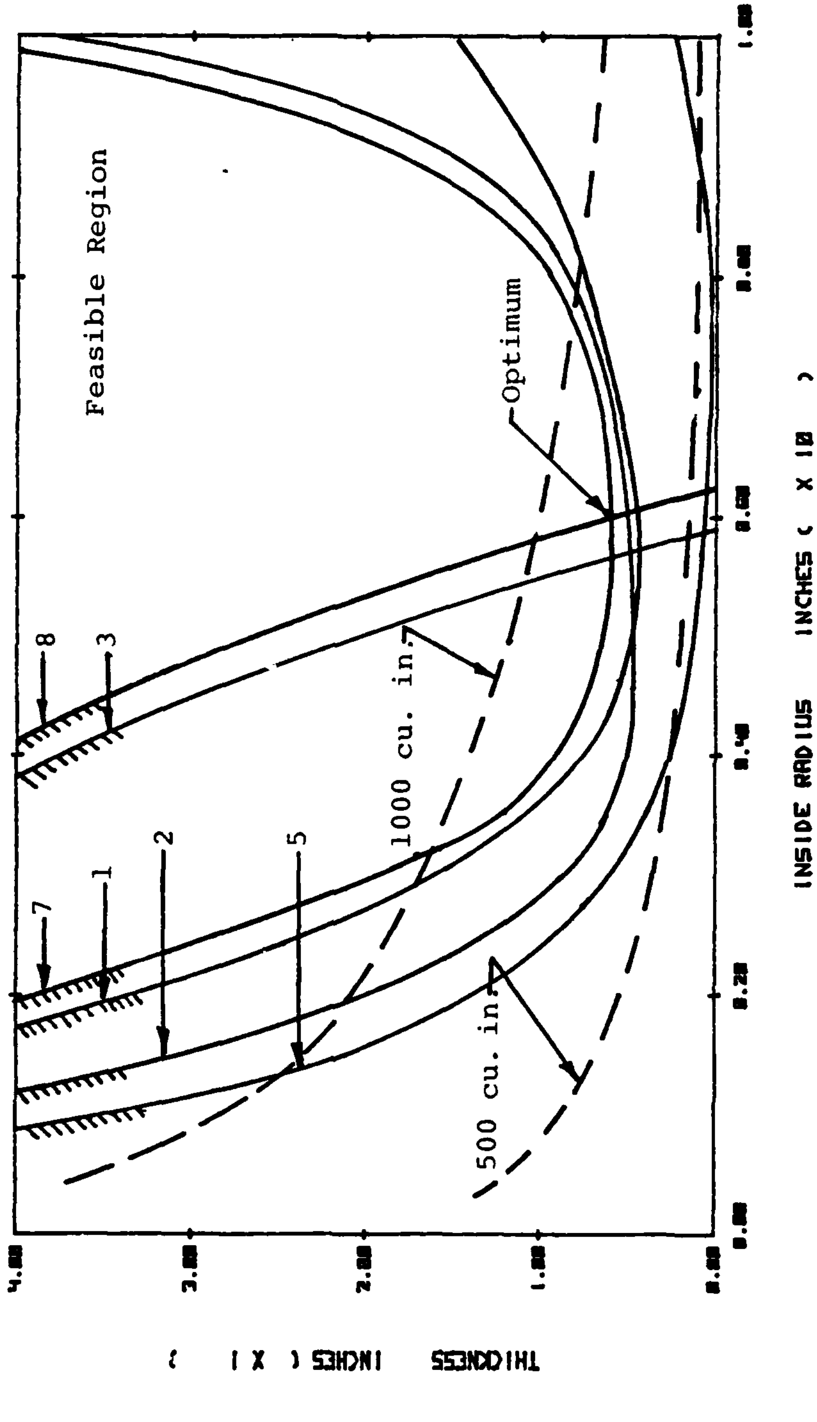
LOADS :

HORSEPOWER	RPM	PRESSURE	AXIAL LOAD
0.15000E 03	0.30000E 04	0.10000E 04	0.0

ANALYSIS RESULTS

TORQUE(1)	0.31500E 04
VOLUME	0.29521E 04
WEIGHT	0.83250E 03
MOMENT OF INERTIA	0.50353E 03
POLAR MOMENT OF INERTIA	0.10071E 04
CRITICAL SPEED	0.60003E 04
CRITICAL AXIAL LOAD	0.65357E 06
CRITICAL BUCKLING TORQUE	0.34401E 08
CRITICAL BUCKLING STRESS	0.16567E 07
DEFLECTION	0.43926E-01
BENDING MOMENT	0.12619E 06
SHEAR FORCE	0.14035E 05
AXIAL STRESS AT A	0.11609E 05
AXIAL STRESS AT B	0.82525E 04
HOOP STRESS	0.12589E 05
TORSIONAL SHEAR STRESS	0.20948E 02
TOTAL SHEAR STRESS	0.11620E 04
MEAN AXIAL STRESS	0.11443E 05
ALTERNATING AXIAL STRESS	0.16609E 03
VON MISES AT MIDSPAN	0.12129E 05
VON MISES AT X=0.	0.12749E 05

EXAMPLE 5



Feasible Region
Optimum
1000 cu. in.
500 cu. in.
8
3
7
1
2
5
INSIDE RADIUS INCHES (X 10)
THICKNESS INCHES (X 1)

EXAMPLE 6. : INITIAL DESIGN

SHAFT DIMENSIONS :

RI = 0.10000E 01 TH = 0.10000E 01 SL = 0.12000E

DESIGN CONDITIONS :

ECCENTRICITY = 0.10000E 00

END CONDITIONS : PINNED-PINNED

LOADS :

HORSEPOWER	RPM	PRESSURE	AXIAL LOAD
0.15000E 03	0.30000E 04	0.0	0.20000E 04

ANALYSIS RESULTS

TORQUE(1)	0.31500E 04
VOLUME	0.11310E 04
WEIGHT	0.31893E 03
MOMENT OF INERTIA	0.11781E 02
POLAR MOMENT OF INERTIA	0.23562E 02
CRITICAL SPEED	0.14757E 04
CRITICAL AXIAL LOAD	0.18764E 06
CRITICAL BUCKLING TORQUE	0.47469E 08
CRITICAL BUCKLING STRESS	0.90549E 07
DEFLECTION	0.24000E 02
BENDING MOMENT	0.56420E 08
SHEAR FORCE	0.62722E 06
AXIAL STRESS AT A	0.95779E 07
AXIAL STRESS AT B	-0.95783E 07
HOOP STRESS	0.25013E 03
TORSIONAL SHEAR STRESS	0.26738E 03
TOTAL SHEAR STRESS	0.13337E 06
MEAN AXIAL STRESS	0.95771E 07
ALTERNATING AXIAL STRESS	0.81250E 03
VON MISES AT MIDSPAN	0.95785E 07
VON MISES AT X=0.	0.23100E 06

EXAMPLE 6. : OPTIMUM DESIGN

SHAFT DIMENSIONS :

RI = 0.63959E 01 TH = 0.36942E-01 SL = 0.12000E

DESIGN CONDITIONS :

ECCENTRICITY = 0.10000E 00

END CONDITIONS : PINNED-PINNED

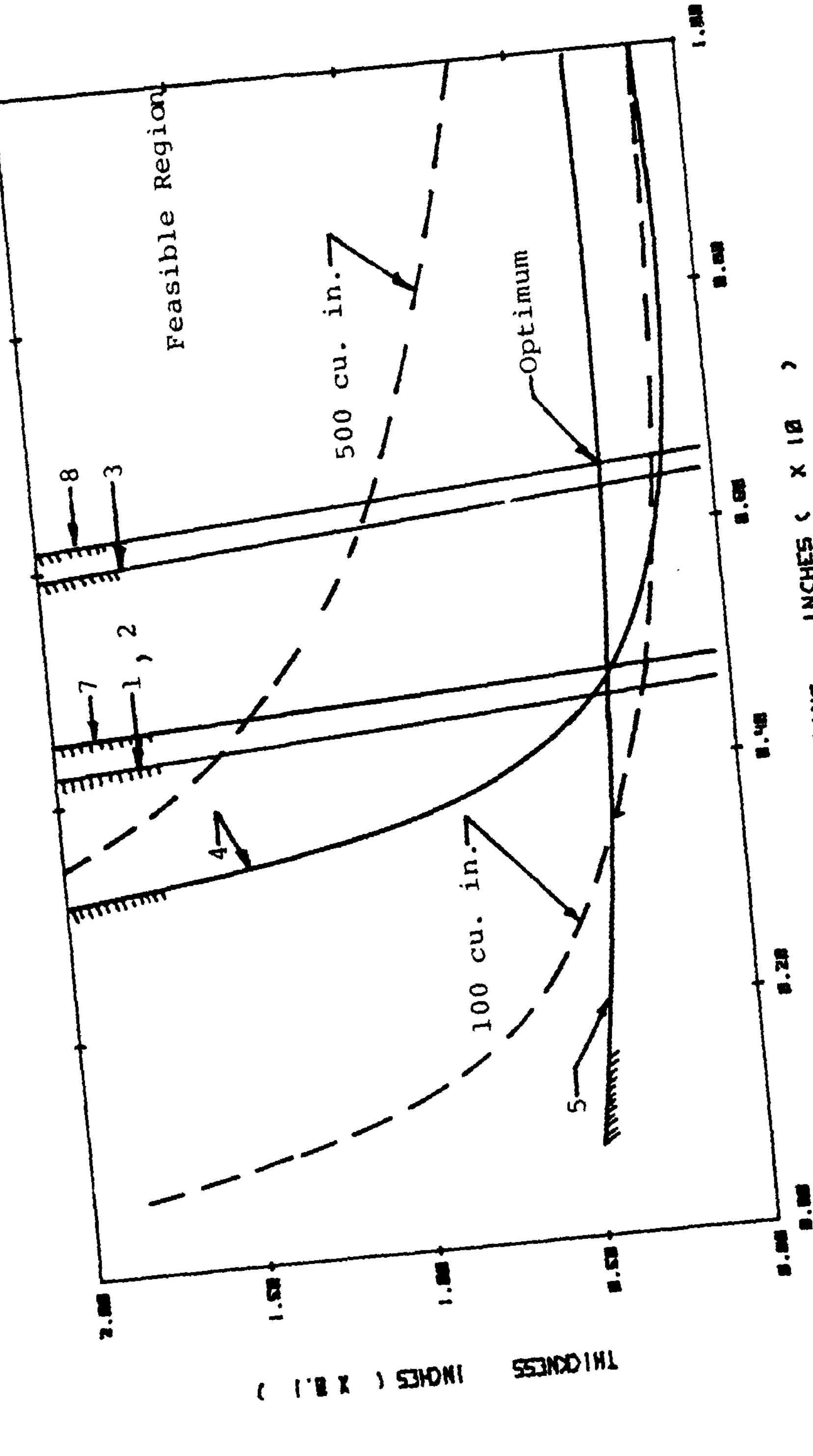
LOADS :

HORSEPOWER	RPM	PRESSURE	AXIAL LOAD
0.15000E 03	0.30000E 04	0.0	0.20000E 04

ANALYSIS RESULTS

TORQUE(1)	0.31500E 04
VOLUME	0.17856E 03
WEIGHT	0.50382E 02
MOMENT OF INERTIA	0.30629E 02
POLAR MOMENT OF INERTIA	0.61258E 02
CRITICAL SPEED	0.60061E 04
CRITICAL AXIAL LOAD	0.39557E 05
CRITICAL BUCKLING TORQUE	0.31490E 05
CRITICAL BUCKLING STRESS	0.10400E 06
DEFLECTION	0.43813E-01
BENDING MOMENT	0.17026E 05
SHEAR FORCE	0.84895E 03
AXIAL STRESS AT A	0.22326E 04
AXIAL STRESS AT B	-0.49193E 04
HOOP STRESS	0.29778E 04
TORSIONAL SHEAR STRESS	0.33079E 03
TOTAL SHEAR STRESS	0.14712E 04
MEAN AXIAL STRESS	0.20739E 04
ALTERNATING AXIAL STRESS	0.15872E 03
VON MISES AT MIDSPAN	0.69313E 04
VON MISES AT X=0.	0.39193E 04

EXAMPLE 6



Feasible Region
500 cu. in.
100 cu. in.
Optimum
8
3
7
1, 2
4
5
INSIDE RADIUS INCHES (X 10)
THICKNESS INCHES (X 0.1)

EXAMPLE 7. : INITIAL DESIGN

SHAFT DIMENSIONS :

RI = 0.10000E 01 TH = 0.10000E 01 SL = 0.12000E

DESIGN CONDITIONS :

ECCENTRICITY = 0.10000E 00

END CONDITIONS : PINNED-PINNED

LOADS :

HORSEPOWER	RPM	PRESSURE	AXIAL LOAD
0.15000E 03	0.30000E 03	0.0	0.0
0.15000E 03	0.30000E 04	0.0	0.20000E 04

ANALYSIS RESULTS

TORQUE(1)	0.31500E 05
TORQUE(2)	0.31500E 04
VOLUME	0.11310E 04
WEIGHT	0.31893E 03
MOMENT OF INERTIA	0.11781E 02
POLAR MOMENT OF INERTIA	0.23562E 02
CRITICAL SPEED	0.14767E 04
CRITICAL AXIAL LOAD	0.18764E 06
CRITICAL BUCKLING TORQUE	0.47469E 08
CRITICAL BUCKLING STRESS	0.90549E 07
DEFLECTION	0.24000E 02
BENDING MOMENT	0.56420E 08
SHEAR FORCE	0.62722E 06
AXIAL STRESS AT A	0.95779E 07
AXIAL STRESS AT B	-0.95783E 07
HOOP STRESS	0.25013E 03
TORSIONAL SHEAR STRESS	0.26738E 03
TOTAL SHEAR STRESS	0.13337E 06
MEAN AXIAL STRESS	0.95771E 07
ALTERNATING AXIAL STRESS	0.81250E 03
VON MISES AT MIDSPAN	0.95785E 07
VON MISES AT X=0.	0.23100E 06

EXAMPLE 7. : OPTIMUM DESIGN

SHAFT DIMENSIONS :

RI = 0.63551E 01 TH = 0.93009E-01 SL = 0.12000E

DESIGN CONDITIONS :

ECCENTRICITY = 0.10000E 00

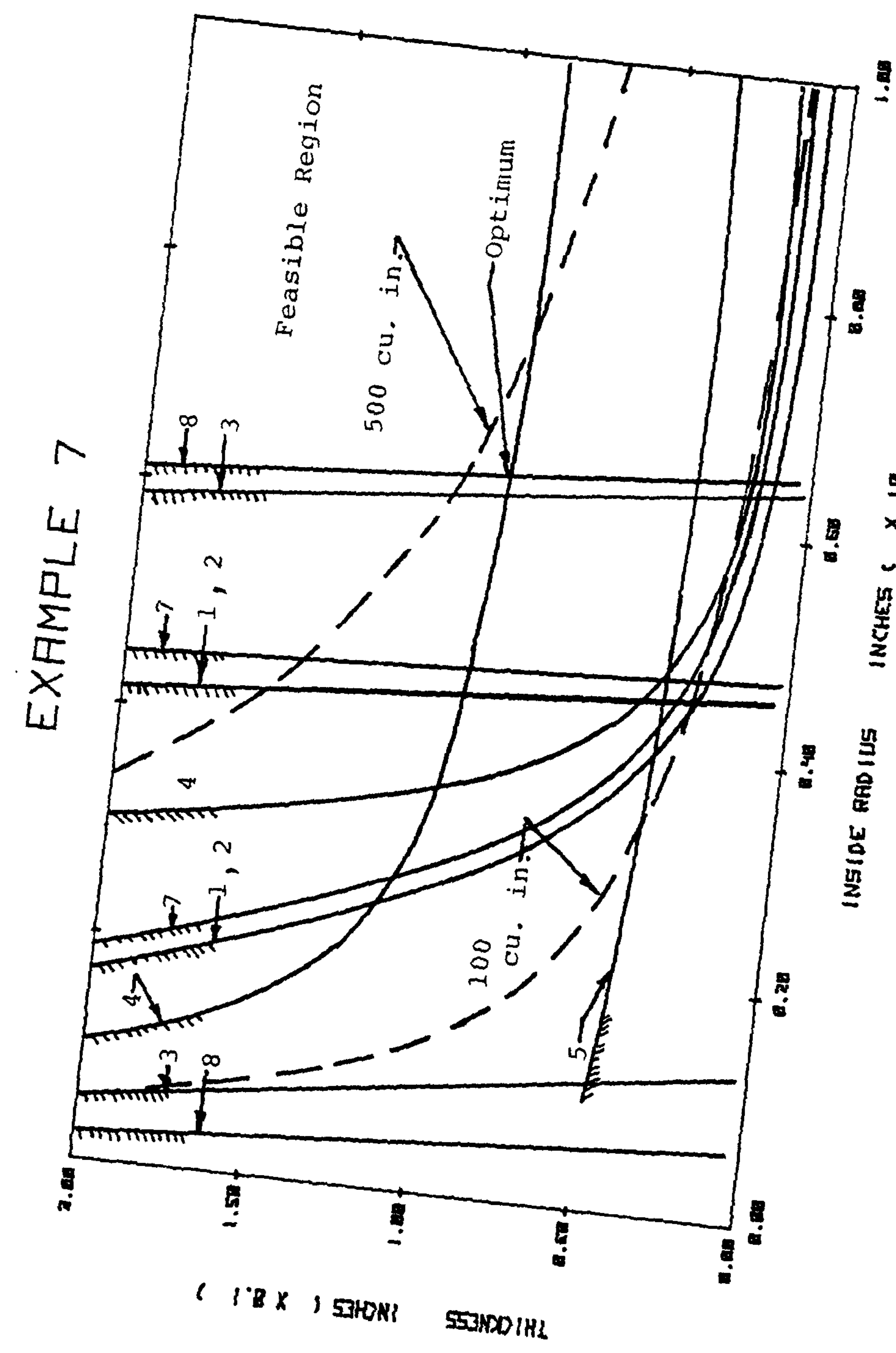
END CONDITIONS : PINNED-PINNED

LOADS :

HORSEPOWER	RPM	PRESSURE	AXIAL LOAD
0.15000E 03	0.30000E 03	0.0	0.0
0.15000E 03	0.30000E 04	0.0	0.20000E 04

ANALYSIS RESULTS

TORQUE(1)	0.31500E 05
TORQUE(2)	0.31500E 04
VOLUME	0.44893E 03
WEIGHT	0.12660E 03
MOMENT OF INERTIA	0.76650E 02
POLAR MOMENT OF INERTIA	0.15332E 03
CRITICAL SPEED	0.60000E 04
CRITICAL AXIAL LOAD	0.99390E 05
CRITICAL BUCKLING TORQUE	0.31571E 06
CRITICAL BUCKLING STRESS	0.26122E 06
DEFLECTION	0.43932E-01
BENDING MOMENT	0.42893E 05
SHEAR FORCE	0.21344E 04
AXIAL STRESS AT A	0.30733E 04
AXIAL STRESS AT B	-0.41425E 04
HOOP STRESS	0.29828E 04
TORSIONAL SHEAR STRESS	0.13248E 03
TOTAL SHEAR STRESS	0.12736E 04
MEAN AXIAL STRESS	0.29136E 04
ALTERNATING AXIAL STRESS	0.15973E 03
VON MISES AT MIDSPAN	0.62021E 04
VON MISES AT X=0.	0.37098E 04


EXAMPLE 7
Feasible Region
Optimum
500 cu. in.
100 cu. in.
8
3
7
1, 2
4
5
INSIDE RADIUS INCHES (X 10)
THICKNESS INCHES (X 0.1)
0.00
0.20
0.40
0.60
0.80
1.00
0.50
1.50
2.00

B. OBSERVATIONS AND CONCLUSIONS

With all other parameters held constant, an increase in speed drives the design to a very large inside radius while reducing the thickness considerably. This trend is so strong that a reasonable lower bound on the thickness must be given to prevent numerical ill-conditioning. The introduction of eccentricity increases the effect of rotation on the stresses and the optimization produces slightly larger values for the design variables. This effect is reduced, however, by the constraint on the deflection.

As the internal pressure is increased, the tendency is to produce an optimized radius that is smaller, and a larger thickness. This may be explained by the direct effect of the internal pressure on the hoop stress which in turn would require a larger thickness to support the higher stress.

The effect of axial load is reflected on the deflection. Again, if very small deflections are allowed, the axial load effects are reduced. The major parameter that is affected is the axial stress.

A notable characteristic shown by the two variable function space diagrams is the tendency of the strength constraints to curl back as the radius is increased. This signifies the existence of an upper limit on size for a given design load. The increase in mass, especially at high speeds, increases the stresses considerably leading to violation of

the strength constraints. This phenomenon is primarily a result of the eccentricity of the mass but also is due to the static weight of the shaft itself.

It is also noted that these strength constraints generally run close to each other in an almost parallel manner. The dynamic or fatigue constraint maintains a position such that coming from the feasible range towards the infeasible area, it is activated before the static constraints are encountered. In effect, the latter constraints are redundant in rotating shafts. However, these are kept in the analysis to allow solution of static problems.

The versatility of the program should not be lost in the emphasis on drive shaft applications. With the appropriate loadings, one may use it to solve structural problems like columns, beams and pressure vessel designs. The objective function need not be confined to the volume. For example, given a specific initial dimensions, one may minimize the Young's modulus to determine whether or not a certain material may be utilized; or to determine how low the modulus could get before the shaft fails. If the outer dimension is critical to the design, one may set a specific value on the diameter and let all other parameters vary during optimization.

In summary, numerical optimization provides an efficient and effective way of improving shaft designs. The analysis

used in this exercise is a first step when compared to the boundless refinements in techniques that are possible.

V. RECOMMENDATIONS FOR FUTURE INVESTIGATIONS

The study has shown the feasibility of using numerical optimization techniques in the design of drive shafts within the limitations imposed in the analysis. Further studies on the same design field may be pursued by eliminating some of these limitations. For example, the unfeasible region defined by the speed constraint extends only to some discrete distance and a supercritical feasible design area does exist. It will be interesting to develop optimization techniques to accommodate this disjoint design field and to investigate the peculiarities this will introduce to the analysis.

Another investigation may be concerned with shafts with variable thickness and radius along its length. The inclusion of radial stresses in the calculations, the use of finite element methods in three dimensional stress analysis of the problem, and vibration considerations are but some of the aspects of research that may be useful in future analyses of composite materials.

The challenge of including the unique material characteristics of composites in the design will invariably stimulate a lot of young minds. However, the handling of these direction oriented properties in stress analysis have already been investigated [6 through 12], and the tools required to

carry out the program are available. COMAND [13] offers a simplified composite analysis procedure for panels. Such procedures, adapted with the program used in this investigation could result in a simplified approach to composite drive shaft analysis. In this procedure, failure of the shaft is assumed if a single ply fails and the entire analysis is performed in a ply by ply basis. Since the stress elements have to be in a differential level, the composite material is assumed to be homogenous and unidirectionally isotropic at the differential range. It will also be necessary to assume that stresses throughout the thickness of the ply is uniform and equal to the stresses at the outer element of the ply. Analysis may then be performed in the classical procedure, solving for the stresses along the major shaft axes. The material properties to be used in the calculation must be equivalent values derived from the directional properties using transformation of axes equations. References 10 and 13 present procedures to calculate the corresponding material properties, stresses and strains along the major axes of the shaft and the fiber orientation coordinates.

More involved investigations could deal with failure modes of the whole shaft instead of a single ply, three dimensional analysis of plys including transverse shear stresses, delamination problems, thermal effects, and others.

The field of composites also opens up another group of considerations like the effects of stacking different ply orientations at different sequences, unsymmetrical laminates, unequal thicknesses of plys and other factors relevant only to composites.

APPENDIX A

The FORTRAN program used in this investigation is described here. Subroutine ANALIZ provides the basic analysis used in the optimization. It reads the initial design description and calculates the values of the objective function, constraints, and all other parameters necessary to solve the analysis problem. COPES-CONMIN updates the design to minimize/maximize the objective function, iterating until no further improvement in the objective function is possible without violating one of the constraints. This process is shown in the flow diagrams.

The global catalog lists the location, FORTRAN name, mathematical symbol and description of the parameters used in the optimization. These parameters are contained in the labeled COMMOM block, GLOBCM.

There are at least five input cards to describe the initial design. The first card contains the initial dimensions--inside radius, thickness and shaft length. The second card describes the material used. Young's modulus, shear modulus, Poisson's ratio, weight per unit volume, yield strength, ultimate strength and compressive strength are read here. The endurance limit is internally calculated using the endurance limit factors read from the third card.

The size factor, CB, is internally updated as the shaft diameter is varied during optimization.

The fourth and fifth cards specify the design and load conditions. The parameter, LC, allows the user to choose the end conditions applicable to his requirements. Pinned-pinned and clamped-clamped analyses are available while clamped-pinned and cantilever end conditions are yet to be developed. The parameter LCC allows multiple loading design. One can thereby design a shaft that is capable of supporting LCC different load combinations.

The magnitude of mass imbalance may be read as a specified distance from the center of the shaft, EO, or as a specified fraction, EC, of the shaft radius. The safety factor generally used in the analysis is also specified in the fourth card.

The fifth card specifies the horsepower, shaft speed in RPM, internal pressure and axial load for each loading. Additional cards are used up to LCC number of loadings. The torque is calculated internally in inch-pounds. The following table summarizes the required information for subroutine ANALIZ.

TABLE IV

INPUT CARDS

CARD	INFORMATION	FORMAT
1	RI, TH, SC	8F10.2
2	E, G, PR, SPWT, I/S SU, YC	8E10.2
3	CA, CE, CC, CD, CE, CF	8F10.2
4	LC, LCC, EO, EC, SF	2I5, 3F10.2
5	HP, RPM, P, F	8F10.2

Subroutine BEND calculates the bending moment, deflection and shear force at the critical cross-sections of the beam.

SUBROUTINE ANALIZ (ICALC)
FLOW DIAGRAM

<u>ICALC = 1</u>

READ INITIAL DIMENSIONS

READ MATERIAL PROPERTIES

READ DESIGN CONDITIONS

PRINT ALL INPUT

<u>ICALC = 2</u>

CALCULATE GEOMETRIC PARAMETERS

CALCULATE OBJECTIVE FUNCTION

CALL SUBROUTINE BEND

- . CALCULATE MAXIMUM DEFLECTION
- . CALCULATE BENDING MOMENTS
- . CALCULATE SHEAR FORCE

CALCULATE STRESSES AT CRITICAL STRESS ELEMENTS

CALCULATE CONSTRAINTS

<u>ICALC = 3</u>

PRINT ANALYSIS RESULTS

OPTIMIZATION FLOW DIAGRAM

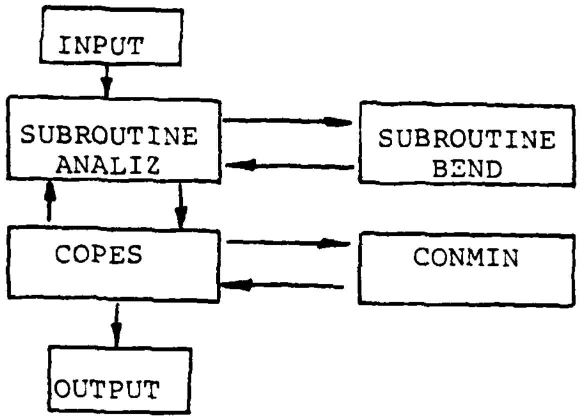


TABLE V

GLOBAL CATALOG OF PARAMETERS

Global Location	FORTRAN Name	Math Symbol	Definition
1	R1	r_i	Inside Radius
2	TH	t	Thickness
3	SL	ℓ	Shaft Length
4	VOL	V	Volume
5-14	TQ(10)	T	Torque
15-24	P(10)	P	Internal Pressure
25-34	F(10)	F	Axial Load
35-44	RDS(10)	ω	Shaft Speed
45	E	E	Young's Modulus
46	G	G	Shear Modulus
47	PR	ν	Poisson's Ratio
48	YS	Sy	Yield Strength (Tension)
49	YC	S_c	Yield Strength (Compression)
50	SU	S_u	Ultimate Strength
51	RO	r_o	Outside Radius
52	SPWT	γ	Weight Per Unit Volume
53	AI	I	Moment of Inertia
54	AJ	J	Polar Moment of Inertia
55	HS	σ_H	Hoop Stress
56	AST	σ_{xA}	Axial Stress at A
57	ASTN	σ_x (min)	Minimum Axial Stress
58-67	ASC(10)	σ_{xB}	Axial Stress at B
68-77	SALT(10)	σ_a, σ_a'	Alternating Axial Stress
78	SMEAN	σ_m	Mean Axial Stress
79	SST	τ_T	Torsional Shear Stress

80	SSV	τ_v	Shear Stress due to Sheer Force
81	SSMX	τ_{max}	Total Shear Stress
82	TCRT	T_c	Critical Torque for Buckling
83	SCSX	σ_c	Critical Stress in Buckling
84-93	VMSI(10)	σ_m	Von Mises (mean)
94-103	VMS2(10)	σ_o'	Von Mises at x = 0
104-113	VMS3(10)	σ_{mid}'	Von Mises at Midspan
114	WM	ω_c	Critical Speed
115	BM	M_{max}	Maximum Bending Moment
116	BMN	M_{min}	Minimum Bending Moment
117	V	V_{max}	Shear Force
118	FC	F_c	Critical Column Buckling Load
119	SF	SF	Safety Factor
120-129	DEL(10)	δ	Deflection
130-239	GC(10,10)	GC	Constraints

```
      SUBROUTINE ANALIZ(ICALC)                                          ANA00010
C        A SUBROUTINE FOR DRIVE SHAFT ANALYSIS TO BE USED IN CONJUNCTION
C         WITH COPES-CONMIN FOR DESIGN OPTIMIZATION.
      COMMON/GLOBCM/RI,TH,SL,VOL,TQ(10),P(10),F(10),RDS(10),E,G,PR,YS,YCANA00020
     *,SU,RO,SPWT,AI,AJ,HS,AST,ASTN,ASC(10),SALT(10),SMEAN,SST,SSV,SSMX,ANA00030
     *TCRT,SCSX,VMS1(10),VMS2(10),VMS3(10),WN,BM,BMN,V,FC,SF,DENOM(10), ANA00040
     *DEL(10),GC(10,10)                                                 ANA00050
      DIMENSION HP(10),RPM(10)                                          ANA00060
      IF(ICALC.GT.1) GO TO 10                                           ANA00070
C-------------INPUT                                                     ANA00080
      GG = 386.                                                         ANA00090
      PI=3.14159265                                                     ANA00100
C        READ INITIAL DIMENSIONS                                        ANA00110
C              RI = INSIDE RADIUS                                       ANA00120
C              TH = THICKNESS                                           ANA00130
C              SL = SHAFT LENGTH                                        ANA00140
C              RO = OUTSIDE RADIUS                                      ANA00150
      READ(5,100) RI,TH,SL                                              ANA00160
C        READ  THE MATERIAL PROPERTIES                                  ANA00170
C              E= YOUNG'S MODULUS                                       ANA00180
C              G = SHEAR MODULUS                                        ANA00190
C              PR = POISSON'S RATIO                                     ANA00200
C              SPWT = WEIGHT PER UNIT VOLUME                            ANA00210
C              YS = YIELD STRENGTH                                      ANA00220
C              SU = ULTIMATE STRENGTH                                   ANA00230
C              SE = ENDURANCE LIMIT                                     ANA00240
C              ENDURANCE LIMIT FACTORS                                  ANA00250
C              CA = SURFACE FACTOR                                      ANA00260
C              CB = SIZE FACTOR                                         ANA00270
C              CC = RELIABILITY FACTOR                                  ANA00280
C              CD = TEMPERATURE FACTOR                                  ANA00290
C              CE = STRESS CONCENTRATION FACTOR                         ANA00300
C              CF = MISCELLANEOUS EFFECTS                               ANA00310
      READ(5,110) E,G,PR,SPWT,YS,SU,YC                                  ANA00320
      READ(5,111)CA,CB,CD,CE,CF,CC                                      ANA00330
C        READ THE LOAD CONDITIONS                                       ANA00340
C              LC = END CONDITIONS                                      ANA00350
C                   1 = PINNED-PINNED                                   ANA00360
C                   2 = CLAMPED-CLAMPED                                 ANA00370
C                   3 = CLAMPED-PINNED                                  ANA00380
C                   4 = CLAMPED-FREE                                    ANA00390
C              LCC = NUMBER OF LOADINGS                                 ANA00400
C              EO = INITIAL ECCENTRICITY                                ANA00410
C              EC = ECCENTRICITY FACTOR WITH RESPECT TO OUTER RADIUS    ANA00420
C              HP = HORSEPOWER                                          ANA00430
C              F = AXIAL LOAD                                           ANA00440
C              P = INTERNAL PRESSURE                                    ANA00450
C              RPM = SHAFT SPEED                                        ANA00460
```

```
      READ(5,120)LC,LCC,EO,EC,SF                                    ANA00470
      DO 5 J=1,LCC                                                  ANA00480
      READ(5,121) HP(J),RPM(J),P(J),F(J)                            ANA00490
      TQ(J)=63000.*HP(J)/RPM(J)                                     ANA00500
      RDS(J) = PI*RPM(J)/30.                                        ANA00510
    5 CONTINUE                                                      ANA00520
C        PRINT ALL INPUT                                            ANA00530
      WRITE(6,130)                                                  ANA00540
      WRITE(6,132) RI,TH,SL                                         ANA00550
      WRITE(6,134)                                                  ANA00560
      WRITE(6,136)E,G,PR,SPWT                                       ANA00570
      WRITE(6,135) YS,YC                                            ANA00580
      WRITE(6,138) LC                                               ANA00590
      DO 6 J=1,LCC                                                  ANA00600
      WRITE(6,141)J                                                 ANA00610
      WRITE(6,140) HP(J),RPM(J),P(J),F(J),TQ(J)                     ANA00620
    6 CONTINUE                                                      ANA00630
      RETURN                                                        ANA00640
C        ANALYZE                                                    ANA00650
   10 IF(ICALC.GT.2) GO TO 20                                       ANA00660
      RO = RI+TH                                                    ANA00670
      ECC=EO+EC*RO                                                  ANA00680
      IF(RO.LE.1.0.AND.RO.GT.0.15) CB=0.85                          ANA00690
      IF(RO.GT.1.) CB=0.75                                          ANA00700
      SE=0.5*SU*CA*CB*CD*CE*CF*CC                                   ANA00710
      VOL=PI*(RO**2-RI**2)*SL                                       ANA00720
      AREA=(RO**2-RI**2)*PI                                         ANA00730
      WT=VOL*SPWT                                                   ANA00740
      WL=WT/SL                                                      ANA00750
C        POLAR MOMENT OF INERTIA                                    ANA00760
      AJ=PI*(RO**4-RI**4)/2.                                        ANA00770
C        MOMENT OF INERTIA                                          ANA00780
      AI=PI*(RO**4-RI**4)/4.                                        ANA00790
C          SLENDERNESS RATIO                                        ANA00800
      SRA = 2.*SL/SQRT(RO**2+RI**2)                                 ANA00810
C          TRANSITION FROM JOHNSON TO EULER                         ANA00820
      Q = 1.                                                        ANA00830
      IF(LC.GT.1) Q=4.                                              ANA00840
      IF(LC.GT.2) Q=2.                                              ANA00850
      IF(LC.GT.3) Q=0.25                                            ANA00860
      SRJ = SQRT(2.*PI**2*Q*E/YC)                                   ANA00870
      IF(SRA.GT.SRJ) GO TO 7                                        ANA00880
      B = (YC/(2.*PI))**2/(Q*E)                                     ANA00890
      FC = AREA*(YC-B*SRA**2)                                       ANA00900
      GO TO 8                                                       ANA00910
    7 FC = AREA*Q*E*PI**2/SRA**2                                    ANA00920
    8 CONTINUE                                                      ANA00930
C          CRITICAL SPEED                                           ANA00940
```

```
      QQ = PI**2                                                        ANA00950
      PP = F(J)*(SL/PI)**2                                              ANA00960
      IF(LC.GT.1) GO TO 12                                              ANA00970
      GO TO 15                                                          ANA00980
   12 QQ = 22.68                                                        ANA00990
      PP = F(J)*0.02533*SL**2                                           ANA01000
   15 ROOT = SQRT((E*AI-PP)/((WL/GG)*SL**4))                            ANA01010
      WN = QQ*ROOT                                                      ANA01020
C              CRITICAL TORQUE FOR BUCKLING                             ANA01030
      TCRT = 1.48*E*SQRT(RI*TH**5)/((1.-PR**2)**0.75)                   ANA01040
C          CRITICAL COMPRESSIVE STRESS FOR BUCKLING                     ANA01050
      SCSX = E*TH/(RO*SQRT(3.*(1.-PR**2)))                              ANA01060
      DO 500 J=1,LC                                                     ANA01070
      CALL BEND(LC,SL,WL,E,AI,ECC,RDS(J),F(J),BMN,BM,V,DJ)              ANA01080
      DEL(J) = DJ                                                       ANA01090
C         CALCULATE THE CRITICAL STRESSES                               ANA01100
C         HOOP STRESS                                                   ANA01110
      HS=2.*RI**2*P(J)/(RO**2-RI**2)+ SPWT*(RO*RDS(J))**2*(3.+PR)*(1.+(1ANA01120
     *.-PR)/(3.+PR) *(RI/RO)**2)/(4.*GG)                                ANA01130
C         TENSILE STRESS AXIAL                                          ANA01140
      AST=P(J)*RI/TH+BM*RO/AI-F(J)/AREA                                 ANA01150
      ASTN = P(J)*RI/TH+BMN*RO/AI-F(J)/AREA                             ANA01160
C         COMPRESSIVE STRESS AXIAL                                      ANA01170
      ASC(J)=P(J)*RI/TH-BM*RO/AI-F(J)/AREA                              ANA01180
      AXE = AST                                                         ANA01190
      IF(ABS(ASC(J)).GT.AST) AXE = ASC(J)                               ANA01200
C         MEAN STRESS AXIAL                                             ANA01210
      SMEAN=(AST+ASTN)/2.                                               ANA01220
C         ALTERNATING STRESS AXIAL                                      ANA01230
      SALT(J)=(AST-ASTN)/2.                                             ANA01240
C         SHEAR STRESS DUE TO TORSION                                   ANA01250
      SST=TQ(J)*RO/AJ                                                   ANA01260
C         SHEAR STRESS DUE TO VERTICAL SHEAR FORCE                      ANA01270
      SSV=2.*V/AREA                                                     ANA01280
C         MAXIMUM SHEAR STRESS                                          ANA01290
      SSMX=SST+SSV                                                      ANA01300
C         VON MISES STRESS                                              ANA01310
      VMS1(J)=SQRT(HS**2-HS*SMEAN+SMEAN**2+3.*SST**2)                   ANA01320
      VMS2(J)=SQRT(HS**2+3.*SSMX**2)                                    ANA01330
      VMS3(J) = SQRT(HS**2-HS*AXE+AXE**2+3.*SST**2)                     ANA01340
      DENOM(J)=VMS1(J)*(SALT(J)/VMS1(J)+SE/SJ)                          ANA01350
C           MAXIMUM DEFLECTION                                          ANA01360
      DMAX = 0.005*SL/12.                                               ANA01370
C         CALCULATE THE CONSTRAINTS                                     ANA01380
      GC(1,J) = VMS3(J)*SF/YS-1.                                        ANA01390
      GC(2,J) = VMS2(J)*SF/YS-1.                                        ANA01400
      GC(3,J) = DEL(J)/DMAX-1.                                          ANA01410
      GC(4,J) = TQ(J)*10./TCRT-1.                                       ANA01420
```

```
      GC(5,J) = ABS(ASC(J))*10./SCSX-1.                                 ANA01430
      GC(6,J) = F(J)*SF/FC-1.                                           ANA01440
      GC(7,J) = DENOM(J)*4./SE-1.                                       ANA01450
      GC(8,J) = RDS(J)*SF/WN-1.                                         ANA01460
      WNRPM = WN*30./PI                                                 ANA01470
  500 CONTINUE                                                          ANA01480
      RETURN                                                            ANA01490
C            PRINT ALL OUTPUT                                           ANA01500
   20 WRITE(6,131)                                                      ANA01510
      WRITE(6,666)                                                      ANA01520
      WRITE(6,132) RI,TH,SL                                             ANA01530
      WRITE(6,501)                                                      ANA01540
      WRITE(6,242) ECC                                                  ANA01550
      IF(LC.EQ.1) WRITE(6,502)                                          ANA01560
      IF(LC.EQ.2) WRITE(6,503)                                          ANA01570
      WRITE(6,504)                                                      ANA01580
      DO 22 J=1,LCC                                                     ANA01590
      WRITE(6,602) HP(J),RPM(J),P(J),F(J)                               ANA01600
   22 CONTINUE                                                          ANA01610
      DO 23 J=1,LCC                                                     ANA01620
      WRITE(6,505) J,TQ(J)                                              ANA01630
   23 CONTINUE                                                          ANA01640
      WRITE(6,506) VOL,WT,AI,AJ                                         ANA01650
      WRITE(6,507) WNRPM,FC,TCRT,SCSX                                   ANA01660
      WRITE(6,508) DEL(J),BM,V                                          ANA01670
      WRITE(6,509) AST,ASC(J),HS,SST,SSMX                               ANA01680
      WRITE(6,600) SMEAN,SALT(J)                                        ANA01690
      WRITE(6,601) VMS3(J),VMS2(J)                                      ANA01700
  100 FORMAT(8F10.2)                                                    ANA01710
  110 FORMAT(8E10.2)                                                    ANA01720
  111 FORMAT(8F10.2)                                                    ANA01730
  120 FORMAT(2I5,3F10.2)                                                ANA01740
  121 FORMAT(8F10.2)                                                    ANA01750
  130 FORMAT(1H1,20X,26HINITIAL DESIGN INFORMATION)                     ANA01760
  131 FORMAT(1H1,20X,16HANALYSIS RESULTS)                               ANA01770
  132 FORMAT(/10X,4HRI =,E12.5,2X,4HTH =,E12.5,2X,4HSL =,E12.5)         ANA01780
  133 FORMAT(/10X,8HVOLUME =,E12.5//10X,8HWEIGHT =,F12.5,//10X,19HMOMENTANA01790
     * OF INERTIA =,E12.5,//10X,25HPOLAR MOMENT OF INERTIA =,E12.5)     ANA01800
  134 FORMAT(//20X,19HMATERIAL PROPERTIES)                              ANA01810
  136 FORMAT(/10X,3HE =,E12.5,2X,3HG =,E12.5,2X,4HPR =,E12.5,2X,8HSP.WT.ANA01820
     * =,E12.5)                                                         ANA01830
  135 FORMAT(/10X,24HYIELD STRENGTH TENSILE =,E12.5,//10X,22HYIELD STRENANA01840
     *GTH COMP. =,E12.5)                                                ANA01850
  137 FORMAT(//10X,17HENDURANCE LIMIT =,E12.5,//10X,19HULTIMATE STRENGTHANA01860
     * =,E12.5)                                                         ANA01870
  138 FORMAT(//20X,13HEND CONDITION,I3)                                 ANA01880
  140 FORMAT(/10X,4HHP =E12.5,2X,5HRPM =,E12.5,2X,3HP =,E12.5,2X,4HF=   ANA01890
     *,E12.5,//10X,8HTORQUE =,E12.5)                                    ANA01900
```

```
141 FORMAT(/20X,11HLOADING NO.,I3)                                   ANA01910
666 FORMAT(/9X,18HSHAFT DIMENSIONS :)                                ANA01920
501 FORMAT(/9X,19HDESIGN CONDITIONS :)                               ANA01930
502 FORMAT(/9X,30HEND CONDITIONS : PINNED-PINNED)                    ANA01940
503 FORMAT(/9X,32HEND CONDITIONS : CLAMPED-CLAMPED)                  ANA01950
504 FORMAT(/9X,7HLOADS :,//9X,10HHORSEPOWER,T22,3HRPM,T35,8HPRESSURE, ANA01960
   *T48,10HAXIAL LOAD)                                               ANA01970
602 FORMAT(/9X,4(E12.5,1X))                                          ANA01980
505 FORMAT(/9X,7HTORQUE(,I1,1H),T40,E12.5)                           ANA01990
506 FORMAT(/9X,6HVOLUME,T40,E12.5,//9X,6HWEIGHT,T40,E12.5,//9X,      ANA02000
   *17HMOMENT OF INERTIA,T40,E12.5,//9X,23HPOLAR MOMENT OF INERTIA,  ANA02010
   *T40,E12.5)                                                       ANA02020
507 FORMAT(/9X,14HCRITICAL SPEED,T40,E12.5,//9X,19HCRITICAL AXIAL LOADANA02030
   *,T40,E12.5,//9X,24HCRITICAL BUCKLING TORQUE,T40,E12.5,//9X,24HCRITANA02040
   *ICAL BUCKLING STRESS,T40,E12.5)                                  ANA02050
508 FORMAT(/9X,10HDEFLECTION,T40,E12.5,//9X,14HBENDING MOMENT,T40,E12.ANA02060
   *5,//9X,11HSHEAR FORCE,T40,E12.5)                                 ANA02070
509 FORMAT(/9X,17HAXIAL STRESS AT A,T40,E12.5,//9X,17HAXIAL STRESS AT ANA02080
   *B,T40,E12.5,//9X,11HHOOP STRESS,T40,E12.5,//9X, 22HTORSIONAL SHEARANA02090
   * STRESS,T40,E12.5,//9X,18HTOTAL SHEAR STRESS,T40,E12.5)          ANA02100
600 FORMAT(/9X,17HMEAN AXIAL STRESS,T40,E12.5,//9X,24HALTERNATING AXIAANA02110
   *L STRESS,T40,E12.5)                                              ANA02120
601 FORMAT(/9X,20HVON MISES AT MIDSPAN,T40,E12.5,//9X,17HVON MISES AT ANA02130
   *X=0.,T40,E12.5)                                                  ANA02140
242 FORMAT(/10X,14HECCENTRICITY =,E12.5)                             ANA02150
    RETURN                                                           ANA02160
    END                                                              ANA02170
```

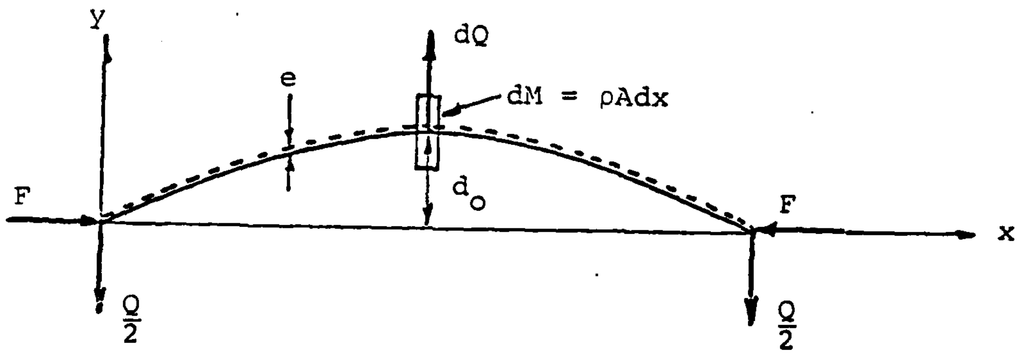
```
      SUBROUTINE BEND(LC,SL,WL,E,AI,ECC,RDS,F,BMN,BM,V,DJ)
C        SUBROUTINE TO CALCULATE THE BENDING MOMENT AND  SHEAR FORCE
C        AT THE CRITICAL CROSS SECTION GIVEN THE END CONDITIONS.
C--------INPUT
C                 LC = END CONDITIONS
C                      1 = PINNED-PINNED
C                      2 = CLAMPED-CLAMPED
C                      3 = CLAMPED-PINNED
C                      4 = CLAMPED-FREE
C                 SL = SHAFT LENGTH
C                 WL = WEIGHT PER UNIT LENGTH
C                 E = YOUNG'S MODULUS OF ELASTICITY
C                 AI = MOMENT OF INERTIA
C                 ECC = ECCENTRICITY
C                 RDS = SHAFT SPEED IN RADIANS PER SECOND
C                 F = AXIAL FORCE
C--------OUTPUT
C                  BMN = MINIMUM BENDING MOMENT AT CRITICAL CROSS SECTION
C                 BM = MAXIMUM BENDING MOMENT AT CRITICAL CROSSSECTION
C                 V = SHEAR FORCE AT A CRITICAL CROSSSECTION
C                  DJ = MAXIMUM DEFLECTION AT CRITICAL CROSS SECTION
      PI = 3.14159265
      GG = 386.
      C1 = WL*RDS**2/GG
      DLIM = 0.2*SL
      DJ = DLIM
      IF(LC.GT.1) GO TO 10
      DNUM = 384.*(E*AI-C1*(SL/PI)**4-F*(SL/PI)**2)
      IF(DNUM.GE.1.0E-6) DJ = (C1*ECC+WL)*5.*SL**4/DNUM
      IF(DJ.GT.DLIM) DJ = DLIM
      BM= C1*(ECC/8.+DJ*(SL/PI)**2+F*DJ)+WL*SL**2/8.
      BMN =BM-WL*SL**2/4.
      V = WL*SL/2.+C1*DJ*SL/PI+C1*ECC*SL/2.
      GO TO 40
   10 IF(LC.GT.2) GO TO 20
      DNUM = 384.*E*AI-C1*(SL/PI)**4*72.7045-96.*F*(SL/PI)**2
      IF(DNUM.GE.1.0E-6) DJ = (C1*ECC+WL)*SL**4/DNUM
      IF(DJ.GT.DLIM) DJ = DLIM
      BM = C1*SL/24.*(DJ*SL/2.+3.*DJ/PI**2+ECC*SL)+F*DJ+WL*SL**2/24.
      BMN = BM-WL*SL**2/12.
      V = WL*SL/2.+C1*DJ*SL/4.+C1*ECC*SL/2.
      GO TO 40
   20 IF(LC.GT.3) GO TO 30
C        ALGORITHM FOR CLAMPED-PINNED END CONDITIONS TO BE DEVELOPED
      GO TO 40
   30 CONTINUE
C        ALGORITHM FOR CLAMPED-FREE END CONDITIONS TO BE DEVELOPED
      BM=WL*SL**2/2.+F*DELI+FRDS*SL/2.
```

```
   40 CONTINJE                                                          ANA02660
      RETURN                                                            ANA02670
      END                                                               ANA02680
```

APPENDIX B

The derivation of the equations for the deflection, bending moments and shear force at the critical cross-sections of the shaft are presented here. Pinned-pinned and fixed-fixed end conditions are analyzed.

1. Pinned-Pinned



Let $y = d_o \sin \frac{\pi x}{\ell}$ (1.1)

$$dQ = dM\ (y+e)\ \omega^2$$

$$dQ = \rho A \omega^2\ (y+e)\ dx \tag{1.2}$$

Let $K_1 = \rho A \omega^2$

$$dQ = K_1\ [d_o \sin \frac{\pi x}{\ell} + e]\ dx \tag{1.3}$$

With the effect of weight:

$$dQ = [K_1(d_o \sin \frac{\pi x}{\ell} + e) - K_2]\, dx \qquad (1.4)$$

where $K_2 = \rho Ag$ = numerically equal to weight per unit length

$$Q = \int_0^{\ell} dQ = K_1[2\,\frac{d_o \ell}{\pi} + e\ell] - K_2\,\ell \qquad (1.5)$$

Moment at $x = x_o$

$$M_{xo} = -\frac{Q}{2}(\ell - X_o) + \int_{x_o}^{\ell} (x - x_o)\, dQ \qquad (1.6)$$

$$M_x = -\frac{Q}{2} + K_1\,\ell^2\,(\frac{d_o}{\pi} + \frac{e}{2}) + [\frac{Q}{2} - K_1\ell\,(\frac{d_o}{\pi} + e)]x \qquad (1.7)$$

$$+ \frac{K_1 e}{2}\,x^2 - K_1\,d_o(\frac{\ell}{\pi})^2 \sin \frac{\pi x}{\ell}$$

$$- \frac{K_2}{2}\,x^2 + \frac{K_2 \ell}{2}\,x$$

This represents the moment at any value of x. If the axial load, F, is included, the total bending moment is

$$M_{TOTAL} = M_x + M_F = EIy''$$

$$\text{where } M_F = Fy = F\,(d_o \sin \frac{\pi x}{\ell}) \qquad (1.8)$$

Integrating twice:

$$EIy = \frac{K_1 e}{24}\,[\ell^3 x - 2\ell x^3 + x^4] + K_1\,d_o\,(\frac{\ell}{\pi})^4 \sin \frac{\pi x}{\ell} \qquad (1.9)$$

$$+ F\,d_o\,(\frac{\ell}{\pi})^2 \sin \frac{\pi x}{\ell} - \frac{K_2}{24}\,[\ell^3 x - 2\ell x^3 + x^4]$$

At $x = \ell/_2$, $y = d_o$

$$EId_o - K_1 \ d_o \ (\frac{\ell}{\pi})^4 - Fd_o(\frac{\ell}{\pi})^2 = \frac{5 \ K_1 e \ell^4}{384} - \frac{5 \ K_2 \ \ell}{384}$$

$$d_o = \frac{5\ell^4}{384} \ [\frac{K_1 e - K_2}{EI - K_1(\frac{\ell}{\pi})^4 - F(\frac{\ell}{\pi})^2}] \qquad (1.10)$$

This is the deflection when the weight effect is subtractive. The maximum deflection occurs when the weight tends to increase the deflection.

$$d_{max} = \frac{5\ell^4}{384} \ [\frac{K_1 e + K_2}{EI - K_1(\frac{\ell}{\pi})^4 - F(\frac{\ell}{\pi})^2}] \qquad (1.11)$$

Substituting (1.10) in (1.7) and (1.8), the bending moment at $x = \ell/_2$ is

$$M_{min} = K_1 \ [\frac{e}{8} + d_o \ (\frac{\ell}{\pi})^2] \ + Fd - \frac{K_2 \ell^2}{8} \qquad (1.12)$$

Again the maximum moment occurs when the weight effect is additive

$$M_{max} = K_1 \ [\frac{e}{8} + d_{max} \ (\frac{\ell}{\pi})^2] \ + Fd + \frac{K_2 \ell 2}{8} \qquad (1.13)$$

From Equation (1.5), the maximum shear force V_{MAX} is equal to $\frac{Q}{2}$ or

$$V_{MAX} = \frac{K_1 \ d_{max} \ell}{\pi} + \frac{K_1 e \ell}{2} + \frac{K_2 \ell}{2} \qquad (1.14)$$

2. Clamped-Clamped

A similar derivation procedure for clamped-clamped ends using a shape function:

$$y = \frac{d_o}{2}\left[1 - \cos\frac{2\pi x}{\ell}\right] \tag{2.1}$$

and

$$dQ = K_1\left[\frac{d_o}{2} - \frac{d_o}{2}\cos\frac{2\pi x}{\ell} + e\right] dx \tag{2.2}$$

$$Q = \int_o^\ell dQ = K_1 \ell\left[\frac{d_o}{2} + e\right] \tag{2.3}$$

Moment at $x = x_o$

$$M = M_o - \frac{Q}{2}x + \int_o^{x_o}(x_o - x)\, dQ$$

$$= M_o - \frac{K_1\ell}{2}\left(\frac{d_o}{2} + e\right) + K_1\int_o^{x_o}\left[\frac{x_o d_o}{2} - \frac{x_o d_o}{2}\cos\frac{2\pi x}{\ell} + x_o e - \frac{x d_o}{2} + \frac{x d_o}{2}\cos\frac{2\pi x}{\ell} - xe\right] dx \tag{2.4}$$

M at any value of x:

$$EIy'' = M_x = M_o - \frac{K_1\ell}{2}\left(\frac{d_o}{2} + e\right)x + \frac{K_1}{2}\left(\frac{d_o}{2} + e\right)x^2 - \frac{K_1 d_o \ell^2}{8\pi^2}\left(1 - \cos\frac{2\pi x}{\ell}\right) \tag{2.5}$$

Integrating once:

$$EIy' = M_o X - \frac{K_1}{2}\left(\frac{d_o}{2} + e\right)\left(\frac{x^3}{3} - \frac{x^2 \ell}{2}\right) - \frac{K_1 d_o \ell^2 x}{8\pi^2} + \frac{K_1 d_o \ell^3}{16\pi^3} \sin \frac{2\pi x}{\ell} + C_1$$

$y' = 0$ at $x = 0,\ \ell/_2$ and $\ell,\ C_1 = 0$

$$M_o = K_1\ \ell^2 \left[\frac{d_o}{24} + \frac{e}{12} + \frac{d_o}{8\pi^2}\right]$$

Integrating once more:

$$\frac{EIy}{K_1} = \left(\frac{d_o}{2} + e\right)\left(\frac{x^4}{24} - \frac{x^3 \ell}{12} + \frac{x^2 \ell^2}{24}\right) - \frac{d_o \ell^4}{32\pi^4} \cos \frac{2\pi x}{\ell} + C_2$$

$y = 0$ at $x = 0$, $C_2 = \dfrac{d_o \ell^4}{32\pi^4}$

$$\frac{EIy}{K_1} = \left(\frac{d_o}{2} + e\right)\left(\frac{x^2}{24}\right)(x - \ell)^2 + \frac{d_o \ell^4}{32\pi^4}\left(1 - \cos \frac{2\pi x}{\ell}\right) \qquad (2.6)$$

$y = d_o$ at $x = \ell/_2$:

$$\frac{EI}{K_1} d_o = \left(\frac{d_o}{2} + e\right)\left(\frac{\ell^2}{96}\right)\left(\frac{\ell^2}{4}\right) + \frac{d_o \ell^4}{16\pi^4}$$

$$\frac{EId_o}{K_1} = \frac{d_o \ell^4}{768} + \frac{e\ell^4}{384} + \frac{d_o \ell^4}{16\pi^4}$$

$$d_o = \frac{K_1 e \ell^4}{384EI - \frac{K_1 \ell^4}{\pi^4}\left(\frac{\pi^4}{2} + 24\right)} \tag{2.7}$$

With axial load F:

Moment due to F $= M_F = M_o - Fy_F = M_o - \frac{Fd_o}{2}\left(1 - \cos\frac{2\pi x}{\ell}\right)$

Integrating once: (2.8)

$$EIy_F' = M_o x - \frac{Fd_o}{2}\left(x - \frac{\ell}{2\pi}\sin\frac{2\pi x}{\ell}\right) + C_1$$

$$y_F' = 0 \text{ at } x = 0, \; C_1 = 0$$

$$y_F' = 0 \text{ at } x = e, \; M_o = \frac{Fd_o}{2}$$

where M_o = Moment at the ends due to the axial load

$$EIy = \frac{Fd_o}{2}\left(\frac{x^2}{2}\right) - \frac{Fd_o}{2}\left[\frac{x^2}{2} + \frac{\ell^2}{4\pi^2}\cos\frac{2\pi x}{\ell}\right] + C_2$$

$$y_F = 0 \text{ at } x = 0, \; C_2 = \frac{Fd_o\ell^2}{8\pi^2}$$

$$y_F = \frac{Fd_o\ell^2}{8\pi^2 EI}\left(1 - \cos\frac{2\pi x}{\ell}\right) \tag{2.9}$$

At $x = \ell/_2$:

$$y_F = \frac{Fd_o\ell^2}{4\pi^2 EI}$$

From Equation (2.6)

$$y_w = \frac{K_1}{EI} \left[\frac{d_o}{2} \left(\frac{x^2}{24}\right) (x - \ell)^2 + e \left(\frac{x^2}{24}\right) (x - \ell)^2 + \frac{d_o \ell^4}{32\pi^4} \left(1 - \cos \frac{2\pi x}{\ell}\right)\right]$$

At $x = \ell/_2$:

$$y_w = \frac{K_1 \ell^4}{384EI} \left[\frac{d_o}{2} + e\right] + \frac{K_1 d_o \ell^4}{16\pi^4 EI}$$

$$d_o = y_F + y_w$$

$$= \left(\frac{F\ell^2}{4\pi^2 EI} + \frac{K_1 \ell^4}{768EI} + \frac{K_1 \ell^4}{16\pi^4 EI}\right) d_o + \frac{K_1 \ell^4 e}{384EI} \qquad (2.10)$$

$$384EI\ d_o = \left(\frac{96F\ell^2}{\pi^2} + \frac{K_1 \ell^4}{2} + \frac{24K_1 \ell^4}{\pi^4}\right) d_o + K_1\ e\ell^4$$

$$d_o = \frac{K_1 e \ell^4}{384EI - K_1\ \ell^4 \left(\frac{1}{2} + \frac{24}{\pi^4}\right) - \frac{96F\ell^2}{\pi^2}} \qquad (2.11)$$

When the effect of weight is considered:

$$d_o = \frac{K_1 e \ell^4 \pm K_2 \ell^4}{384EI - 0.74638K_1 \ell^4 - 96\left(\frac{\ell}{\pi}\right)^2 F} \qquad (2.12)$$

where $K_2 = \rho Ag$, numerically equal to weight per unit length.

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